MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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F. W. DYSON, Esq., M.A., LL.D., F.R.S., PRESIDENT, in the Chair.

Ernest Herbert Beattie, The Observatory, Orlando Avenue, West Point, Mosman, Sydney, New South Wales;
Hector Macpherson jun., Alpine Villa, Currie, Midlothian, Scotland; and
Frederick Slocum, Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Allen Abraham, B.A., Professor of Mathematics, Jaffna College, Vaddukkoddai, Ceylon (proposed by A. V. Jugga Row);
Gerald Victor Adendorff, Electrical Engineer, Transvaal Power Co., P.O. box 2671, Johannesburg, South Africa (proposed by R. T. A. Innes);
David Baxandall, A.R.C.Sc., Science Museum, South Kensington, S.W. (proposed by A. Fowler);
Rev. Septimus Buss, LL.B., Rector of St. Anne and St. Agnes, Gresham Street, E.C., 82 Caversham Road, Camden Road, N.W. (proposed by S. A. Saunder);

Charles Clifford Conroy, A.M., Sc.D., Professor of Astronomy, Los Angeles College, California, U.S.A., P.O. box 826, Los Angeles (proposed by T. J. J. See);

Dudley Eglinton, Walmsley Street, Kangaroo Point, Brisbane, Queensland, Australia (proposed by C. F. Plant);

George James Gibbs, M.Inst.M.E., Associate of the City and Guilds of London Institution (proposed by the Rev. A. L. Cortie);

James Martin Hatfield, Architect and Surveyor, 1 Palewell Park, East Sheen, S.W. (proposed by T. Davies Sewell);

George Hall Hamilton, B.A. Cantab., Professor of Astronomy and Physics, Bellevue College, Bellevue, Nebraska, U.S.A. (proposed by Sir R. S. Ball);

Richard Julius Hoffmann, Tower House, Leigham Court Road, Streatham, S.W. (proposed by W. J. S. Lockyer);

Oliver J. Lee, Yerkes Observatory, Williams Bay, Wisconsin, U.S.A. (proposed by S. W. Burnham);

Captain Robert McFee, Commander R.N.R., Superintendent, Anchor Line Steamers, St. Vincent's Place, Glasgow (proposed by the Rev. James Baikie);

Frederick Poate, Surveyor-General of New South Wales, Sydney, Australia (proposed by T. F. Furber);

William Herbert Steavenson, Ariel Lodge, Cheltenham (proposed by F. W. Dyson); and

Harold Thomson, 37 Larkspur Terrace, Jesmond, Newcastle-upon-Tyne (proposed by the Rev. T. E. R. Phillips).

One hundred and ninety presents were announced as having been received since the last meeting, including, amongst others:—

H. Andoyer, Nouvelles Tables Trigonométriques Fondamentales (Logarithmes), presented by MM. A. Hermann et fils; Lewis Boss, List of 1059 Standard Stars for 1910, presented by the author; Cowell and Crommelin, Investigation of the motion of Halley's Comet from 1759 to 1910, presented by the Royal Observatory, Greenwich; Sir G. H. Darwin, Scientific papers, vol. iv., presented by the author; Walter Goodacre, A map of the Moon in xxv. sections, with Index, presented by the author; E. S. Grew, The Growth of a Planet, presented by Mr. Lynn; F. W. Henkel, Weather Science, presented by the author; A. R. Hinks, Astronomy, presented by the author; Transactions of the International Union for Solar Research, vol. iii. (fourth Conference), presented by Messrs. Sherratt & Hughes; Adams and Lasby, Investigation of the rotation of the Sun by spectroscopic methods, presented by the Solar Observatory, Mount Wilson; Paris Observatory, Catalogue photographique du Ciel; Coordonnées rectilignes, presented by the Observatory; H. Poincaré, Leçons sur les Hypothèses Cosmogoniques, presented by MM. A. Hermann et fils.

Series of Transparencies of Spectroheliograms, taken at the Meudon Observatory, presented by M. H. Deslandres; Photographs of stellar spectra (eight transparencies) taken at the Lick Observatory, presented by the Observatory; Melbourne Observatory, 41 transparencies of Comet Morehouse and 17 of Halley's Comet taken with the astrographic telescope and 6-inch doublet, presented by the Observatory; Series of lantern slides from photographs of the Eclipse of 1911 April, taken by the Adelaide Observatory Expedition, presented by G. F. Dodwell; Series of transparencies of the Eclipse of 1911 April, taken by the Melbourne Observatory Expedition, presented by Mr Baracchi.

Astrographic Chart: 91 charts from Algiers, Bordeaux, Toulouse, and Paris, presented by the French Minister of Public Instruction.

Typed copies of early scarce numbers of the *Monthly Notices*, presented by Prof. Campbell. Portrait of Prof. Newall for the series of photographs of Presidents of the Society, presented by Prof. Newall.

Mean Areas and Heliographic Latitudes of Sun-spots in the Year 1910.

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those printed in the *Monthly Notices*, vol. lxxi. p. 68, and are deduced from the measurements of photographs taken at the Royal Observatories of Greenwich and of the Cape, and in India at the Kodaikánal Observatory and at Dehra Dûn, North-West Provinces.

Table I. gives the mean daily area of umbræ, whole spots, and faculæ for each synodic rotation of the Sun in the year 1910; and Table II. gives the same particulars for each of the entire years 1901 to 1910 inclusive. The areas are given in two forms; first, projected areas, that is to say, as seen and measured on the photographs, these being expressed as millionths of the Sun's apparent disc; and next, areas as corrected for foreshortening, the areas in this case being expressed in millionths of the Sun's visible hemisphere.

Table III. exhibits for each rotation in the year 1910 the mean daily area of the whole spots (corrected for foreshortening), and the mean heliographic latitude of the spotted areas for spots north and for spots south of the equator, together with the mean heliographic latitude of the entire spotted area, and the mean distance from the equator of all spots; and Table IV. gives the same information for each year as a whole for the years 1901 to 1910 inclusive. Tables II. and IV. are thus in continuation of the similar tables for the years 1874 to 1888 on pp. 381 and 382 of vol. xlix. of the

Monthly Notices, and for the years 1889 to 1902 on pp. 465 and 466 of vol. lxiii., and for the years 1901 to 1909 on pp. 70 and 72 of vol. lxxi.

TABLE I.

No. of Rotation.	Date of Commencement of each Rotation.		No. of Days on which Photographs were taken.	Mean of Daily Areas.					
				Projected.			Corrected for Foreshortening.		
				Umbræ.	Whole Spots.	Faculæ.	Umbræ.	Whole Spots.	Faculæ.
	1910.	d							
753	Jan.	6·97	28	76	377	998	55	285	1045
754	Feb.	3·31	27	180	942	981	120	636	986
755	Mar.	2·65	27	71	374	1106	57	309	1077
756		29·96	27	13	81	734	10	59	727
757	Apr.	26·23	28	83	483	1084	60	344	1037
758	May	23·45	27	15	92	960	14	86	960
759	June	19·66	27	62	325	1153	49	269	1128
760	July	16·86	27	51	283	1023	34	191	963
761	Aug.	13·07	27	35	173	837	36	167	839
762	Sept.	9·32	28	130	736	1182	95	548	1125
763	Oct.	6·60	27	99	546	1203	73	392	1210
764	Nov.	2·89	27	7	40	875	4	23	859
765		30·20	28	10	51	694	8	40	697

TABLE II.

Year.	No. of Days on which Photographs were taken.	Mean of Daily Areas.					
		Projected.			Corrected for Foreshortening.		
		Umbræ.	Whole Spots.	Faculæ.	Umbræ.	Whole Spots.	Faculæ.
1901	359	14	41	23	9	29	29
1902	349	14	86	163	10	62	178
1903	350	67	434	875	51	340	970
1904	363	93	653	1639	67	488	1761
1905	364	230	1637	2433	163	1191	2612
1906	364	158	1047	2207	114	778	2320
1907	365	221	1453	1859	160	1082	1999
1908	366	148	952	2000	106	697	2098
1909	364	171	941	1303	124	692	1353
1910	365	67	357	984	49	264	971

TABLE III.

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Spots NORTH of the Equator.		Spots SOUTH of the Equator.		Mean Heliographic Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
			Mean of Daily Areas.	Mean Heliographic Latitude.	Mean of Daily Areas.	Mean Heliographic Latitude.		
	1910. d			°		°	°	°
753	Jan. 6·97	28	49	5·80	235	10·77	− 7·90	9·91
754	Feb. 3·31	27	14	14·09	622	10·34	− 9·80	10·42
755	Mar. 2·65	27	175	16·34	134	12·01	+ 4·05	14·46
756	29·96	27	20	13·65	39	7·55	− 0·30	9·64
757	Apr. 26·23	28	87	2·53	257	11·69	− 8·08	9·37
758	May 23·45	27	42	7·73	44	10·31	− 1·57	9·06
759	June 19·66	27	151	7·96	118	5·44	+ 2·09	6·86
760	July 16·86	27	1	16·04	190	12·80	− 12·58	12·83
761	Aug. 13·07	27	0	...	167	13·26	− 13·26	13·26
762	Sept. 9·32	28	2	6·36	546	12·26	− 12·20	12·24
763	Oct. 6·60	27	246	6·38	146	9·72	+ 0·40	7·62
764	Nov. 2·89	27	16	9·69	7	11·43	+ 3·31	10·22
765	30·20	28	0	...	40	6·74	− 6·74	6·74

TABLE IV.

Year.	No. of Days on which Photographs were taken.	Spots NORTH of the Equator.		Spots SOUTH of the Equator.		Mean Heliographic Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
		Mean of Daily Areas.	Mean Heliographic Latitude.	Mean of Daily Areas.	Mean Heliographic Latitude.		
			°		°	°	°
1901	359	22	8·59	7	16·27	+ 2·82	10·37
1902	349	42	18·81	21	15·29	+ 7·48	17·64
1903	350	132	18·11	208	21·15	− 5·85	19·94
1904	363	268	16·33	220	16·88	+ 1·37	16·57
1905	364	750	11·66	440	15·55	+ 1·60	13·10
1906	364	539	13·98	239	14·01	+ 5·38	13·99
1907	365	488	10·12	593	13·77	− 2·98	12·12
1908	366	316	10·42	381	10·34	− 0·92	10·38
1909	364	299	9·45	393	9·92	− 1·55	9·71
1910	365	66	8·55	198	11·18	− 6·27	10·53

The rotations in Table I. and Table II. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node at mean

noon of 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days. The dates of the commencement of the rotations are given in Greenwich civil time, reckoning from mean midnight.

The principal features of the record for 1910 are:—

1. A great and sudden falling off in the mean daily spotted area, which in 1910 was only 38 per cent. of its value in 1909. This was the more remarkable, as the mean area for 1909 had been almost exactly the same as for 1908. Comparing the mean area for 1910—viz. 264 millionths of the Sun's visible hemisphere—with those for corresponding years in the preceding cycles, the present cycle would appear to promise to reach its minimum in 1913, or a little earlier.

Year.	Mean Daily Area.	Year of Minimum.	Mean Daily Area.
1875	248	1878	22
1887	179	1889	78
1898	375	1901	29

2. An examination of the results for the separate rotations shows that the year opened with the spot activity on the decline, and that three chief waves were shown by its progress during the year. A maximum in February was followed by a marked minimum in April; a slight recovery and decline took place in May and June; and a maximum in September and October was followed by a great depression lasting to the end of the year.

3. The curve shown by the areas of the faculæ has, as usual, been much smoother than that of the spots, and the falling off, as compared with the record of 1909, has been less considerable; the mean area being 72 per cent. of that of 1909, instead of 38 per cent. as with the spots. As compared with the corresponding years in the preceding cycles, the mean area of the faculæ—971—would appear to place the next minimum later than 1913 rather than earlier.

Year.	Mean Daily Area.	Year of Minimum.	Mean Daily Area.
1874	994	1878	67
1885	579	1889	131
1898	891	1901	29

4. Comparing the whole spots of the two hemispheres, the southern hemisphere has been three times as prolific as the northern; indeed in two rotations the northern hemisphere was quite free from spots, and in two others very nearly so. The preponderance of the southern hemisphere, characteristic of the period of decline from maximum to minimum, was therefore strongly marked.

5. The mean distance from the equator of all spots has shown a slight increase as compared with 1908 and 1909.

6. The Sun was free from spots on 65 days in 1910, as against five in 1909 and six in 1908.

7. In the distribution of spots in latitude, both hemispheres have shown disturbances over the entire zone from lat. 24° to the equator, with two maxima in each; in the northern, at lat. +8° and +18°; in the southern, at lat. −7° and −15°.

8. The number of separate groups of spots showed a great falling off as compared with 1909, but not in the same proportion as the mean daily areas; the groups being 151 in number, instead of 237 as in 1909, or 64 per cent. In other words, there was a great increase in the number of small and short-lived groups; whereas in 1909 these had shown a marked diminution.

9. Of the 151 separate groups in 1910, 45 were in the northern hemisphere and 106 in the southern; or expressed in percentages, 30 north to 70 south. The preponderance of activity in the southern hemisphere as shown by the number of separate groups has steadily increased year by year since 1907, when it first became manifest

Royal Observatory, Greenwich:
1911 *October* 28.

Observations of Jupiter's Eighth Satellite.

(*Communicated by the Astronomer Royal.*)

At its recent opposition, Jupiter was so far south of the equator that the eighth satellite was practically beyond the range of the 30-inch reflector at Greenwich.

The Superintendent of the Helwân Observatory, however, undertook to keep the satellite under observation with the Reynold's 30-inch reflector, and 22 successful photographs were secured on 21 nights. The photographs were sent to Greenwich for measurement and reduction.

It was considered desirable to continue the method employed in former years of projecting the satellite *via* faint comparison stars on to astrographic plates, and H.M. Astronomer at the Cape of Good Hope was good enough to have photographs taken with the Cape astrographic telescope covering the region in which the path of Jupiter lay.

The error of the tabular place of Jupiter for comparison with the deduced R.A. and Dec. of J. VIII. was determined from five photographs taken with the Thompson 26-inch photographic refractor (June 3, 7 (two), 10, and 18), the positions of the reference stars being derived from measures on a photograph taken with the Greenwich 13-inch astrographic refractor.

The positions of the stars used in determining the constants of reference plates for the photographs February 4 to May 4 were taken from the A.G. (Washington) Catalogue.

The positions of the stars used on reference plates for the photographs May 20 to June 27, and the reference plate used in determining the error of Jupiter's tabular place were kindly

supplied in manuscript by Professor Pickering from the sheets of the A.G. (Harvard) Catalogue, now in the press.

The correction to the tabular place of Jupiter was found to be

R.A. $+0^{s}\cdot01$ Dec. $-0''\cdot1$

but has not been applied.

The observed places are compared with the ephemeris computed by Dr. Crommelin in continuation of Cowell, Crommelin, and Davidson's orbit (*Monthly Notices*, vol. lxix., No. 5).

The excellent series of photographs of the satellite with exposures varying from one hour to two hours and a half were taken by Mr. Knox-Shaw, or, in his absence, by Mr. Hurst. The measurement and reduction of the photographs were made by Mr. Davidson.

OBSERVATIONS OF SATELLITE VIII.

Date and G.M.T.	Exposure.	Apparent R.A.	Apparent Dec.	Sat. VIII.—Jupiter. R.A.	Sat. VIII.—Jupiter. Dec.	Tab.—Obs. R.A.	Tab.—Obs. Dec.
1910. d h m s	m	h m s	° ′ ″	m s	′ ″	s	″
Feb. 4 14 16 8	120	14 43 41·72	−15 13 2·5	−2 33·38	−26 37·4	+0·77	−13·3
6 14 7 13	120	14 44 20·22	15 16 54·2	2 28·69	28 19·0	0·76	16·5
24 13 2 4	100	14 48 6·46	15 42 34·7	1 41·91	44 3·3	0·91	11·8
Mar. 2 12 51 19	74	14 48 30·61	15 47 5·8	1 25·12	49 10·4	1·02	11·9
9 13 54 49	60	14 48 25·12	15 49 45·8	1 4·84	55 3·0	0·97	12·1
26 10 39 34	63	14 45 47·14	15 44 52·0	0 16·65	68 14·1	1·10	11·9
28 12 21 53	90	14 45 14·61	15 43 13·6	−0 10·81	69 45·5	1·05	11·3
Apr. 3 12 24 8	86	14 43 25·84	15 37 13·7	+0 5·76	73 57·1	1·06	11·6
20 8 35 34	60	14 36 50·73	15 12 31·8	0 49·68	84 13·5	1·14	8·8
26 7 43 59	60	14 34 10·52	15 1 41·4	1 4·50	87 11·7	0·97	10·3
26 8 49 48	60	14 34 9·14	15 1 37·2	1 4·49	87 14·0	1·09	9·3
May 1 8 31 5	80	14 31 51·80	14 52 5·5	1 16·34	89 26·8	1·08	8·5
4 12 2 40	60	14 30 25·24	14 45 58·1	1 23·82	90 42·9	0·93	8·1
20 8 52 0	120	14 23 30·24	14 15 44·6	1 59·05	95 31·2	1·13	7·1
25 9 6 0	105	14 21 35·12	14 7 6·8	2 9·94	96 32·5	1·05	5·1
27 7 38 0	100	14 20(52·38)	14 3(53·1)	2(13·03)	96(47·3)	(2·11)	(9·6)
30 8 23 0	100	14 19 52·11	13 59 14·8	2 20·72	97 15·7	0·88	7·0
June 18 8 21 0	110	14 15 26·67	13 38 56·4	3 1·02	98 22·8	0·62	2·5
21 8 14 0	110	14 15 5·58	13 37 19·8	3 7·10	98 17·6	0·84	4·3
24 8 32 0	135	14 14 50·81	13 36 16·5	3 13·47	98 13·5	0·79	2·1
26 8 15 0	124	14 14 44·34	13 35 47·0	3 17·52	98 4·9	0·91	4·9
27 8 9 0	158	14 14 42·10	−13 35 41·0	+3 19·50	−98 3·9	+1·02	−2·5

Notes.

Feb. 6. A white spot has probably obliterated part of the image.
Feb. 24. Image very diffused.
Mar. 2. Very faint and indefinite.
Mar. 9. Very faint.
May 4. Very indefinite.
May 27. If the time is assumed to be 1 hour in error, *i.e.* $8^{h}\ 38^{m}$ instead of $7^{h}\ 38^{m}$, the residuals would become R.A. $+1^{s}\cdot23$, Dec. $-5''\cdot7$.
June 18. Very elongated in R.A.

Royal Observatory, Greenwich;
1911

On Possible Phase-relations between the Planets and Sun-spot Phenomena. Papers of the I.U.S.R. Computing Bureau. No. VII. By F. J. M. Stratton, M.A., Assistant in Astrophysics, Cambridge Observatory.

§ 1. *Explanatory.*—The following investigation was suggested in the first place by a communication of Mrs. A. S. D. Maunder's (*M.N.R.A.S.*, vol. lxvii., No. 7) noting an apparent connection between the Earth and sun-spot phenomena. The recent publication, by Dr. Schuster, of a paper on "The Influence of Planets on the Formation of Sun-spots" (*Proc. Roy. Soc.*, vol. 85, A, p. 309), suggests the desirability of publishing the following figures partly as an independent confirmation of Dr. Schuster's results and partly as a check on some of his conclusions. The method adopted in the work is closely parallel to that of Dr. Schuster. The Sun's surface was divided into 24 lunes of 15° each, the origin being the meridian passing through a given planet at the instant of any observation. These lunes correspond to hours of what we may call a planet-ruled day, or more simply a planet-day (the day starts when the planet is in the meridian of an observer or spot on the Sun); thus the hours 0 to 6 correspond to points on the Sun whose meridian has already been crossed by a planet (still visible in the sky), *i.e.* to points on the western half of the Sun's disc as viewed from the planet; the hours 6 to 18 give the side of the Sun invisible from the planet, 18 to 24 the eastern half of the visible disc. The sun-spot phenomena have been referred to these lunes or hours.

Fig. 1 gives for convenience of comparison the relative arrangements of Dr. Schuster's lunes and of the hours of this paper. The inner circle refers to Dr. Schuster's work.

The angle between the Sun's equator and the ecliptic is neglected, and the hour-angle of the spot relative to the planet is given by the formula

(heliocentric longitude of spot referred to central meridian
+ heliocentric longitude of Earth – heliocentric longitude of planet).

Only the two planets which are tidally the most effective, Jupiter and Venus,* have been considered; it may be mentioned that the original underlying idea of this investigation was that tidal action might offer the necessary stimulus to upset a delicately balanced condition of equilibrium, and that some similarity might be found in the relations of Jupiter and Venus to the spot-phenomena. The period for which the figures were at first obtained was 1874–1905, and the sources of the information

* See Table *M.N.R.A.S.*, vol. lx. p. 600, the figure for Mercury there has been inadvertently multiplied by 10.

were the Greenwich Photo-heliographic Results. The figures for 1906–1909 became available after the work was first completed, and they have now been included in the investigation. In cases where the additional material shows any marked change in the results, attention has been drawn to the fact below. In general, the conclusions drawn from the earlier work are not affected by the later addition. In all, some 30,000 spot entries have been dealt with, which includes about 2000 first or last appearances near the centre of the Sun's disc, and 1000 single-day spots.

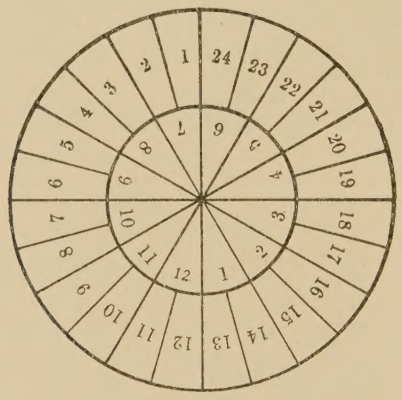

FIG. 1.

§ 2. *Spot-births.*—(*a*) *Jupiter.* In the following table all spots which make their first appearance within 50° of the central meridian of the Sun's disc as viewed from the Earth are entered in the appropriate hour of the Jupiter day. Spots which have been only seen on one day are separated from the rest, and on the suggestion of Mrs. Maunder spots north and south of the solar equator have been separated throughout. This is important, as the time-incidences of the two sets of spots differ very considerably; any agreement in their distribution among the hours of a planet-day is significant, as the accidental inequalities should not be the same in both sets. For Jupiter the interval dealt with is 1874 March 17 to 1909 February 28, or 32 synodic revolutions reckoned from opposition. In Table I. are given the totals for the spot-births and the single-day spots.

TABLE I.

Spot-births and Single-day Spots observed within 50° of the central meridian, arranged in 24 hours of a Jupiter Day.

Hour.	Births.			Single-day Spots.			Combined Total.
	S.	N.	Total.	S.	N.	Total.	
1	48	45	93	18	18	36	129
2	46	44	90	26	23	49	139
3	57	38	95	38	32	70	165
4	46	55	101	25	23	48	149
5	53	32	85	22	20	42	127
6	39	38	77	29	26	55	132
7	55	36	91	30	22	52	143
8	50	54	104	23	28	51	155
9	50	37	87	24	22	46	133
10	40	35	75	24	36	60	135
11	43	48	91	17	18	35	126
12	65	31	96	26	25	51	147
13	50	39	89	22	21	43	132
14	52	39	91	32	37	69	160
15	49	43	92	19	29	48	140
16	44	38	82	36	20	56	138
17	41	44	85	22	23	45	130
18	41	35	76	29	17	46	122
19	46	45	91	17	16	33	124
20	50	42	92	32	23	55	147
21	46	40	86	25	26	51	137
22	57	47	104	32	24	56	160
23	41	41	82	21	33	54	136
24	40	40	80	25	18	43	123
Total	1149	986	2135	614	580	1194	3329
Average	48	41	89	26	24	50	139

Fig. 2 contains graphs representing this and several other tables.

It will be noticed that there are four maxima in the combined totals at 3, 8, 14, and 22 h., corresponding to Dr. Schuster's 8, 10, 1, and 5. Two of these maxima (3, 14) may be traced mainly to the single-day spots, the other two (8, 22) are due mainly to the spots of longer duration.

(*b*) *Venus.* The corresponding figures for Venus are given in Table II. The interval considered is 1874 December 8 to 1908 July 5, 21 synodic revolutions reckoned from inferior conjunction. The same limitations are imposed upon the spots tabulated as in the previous table.

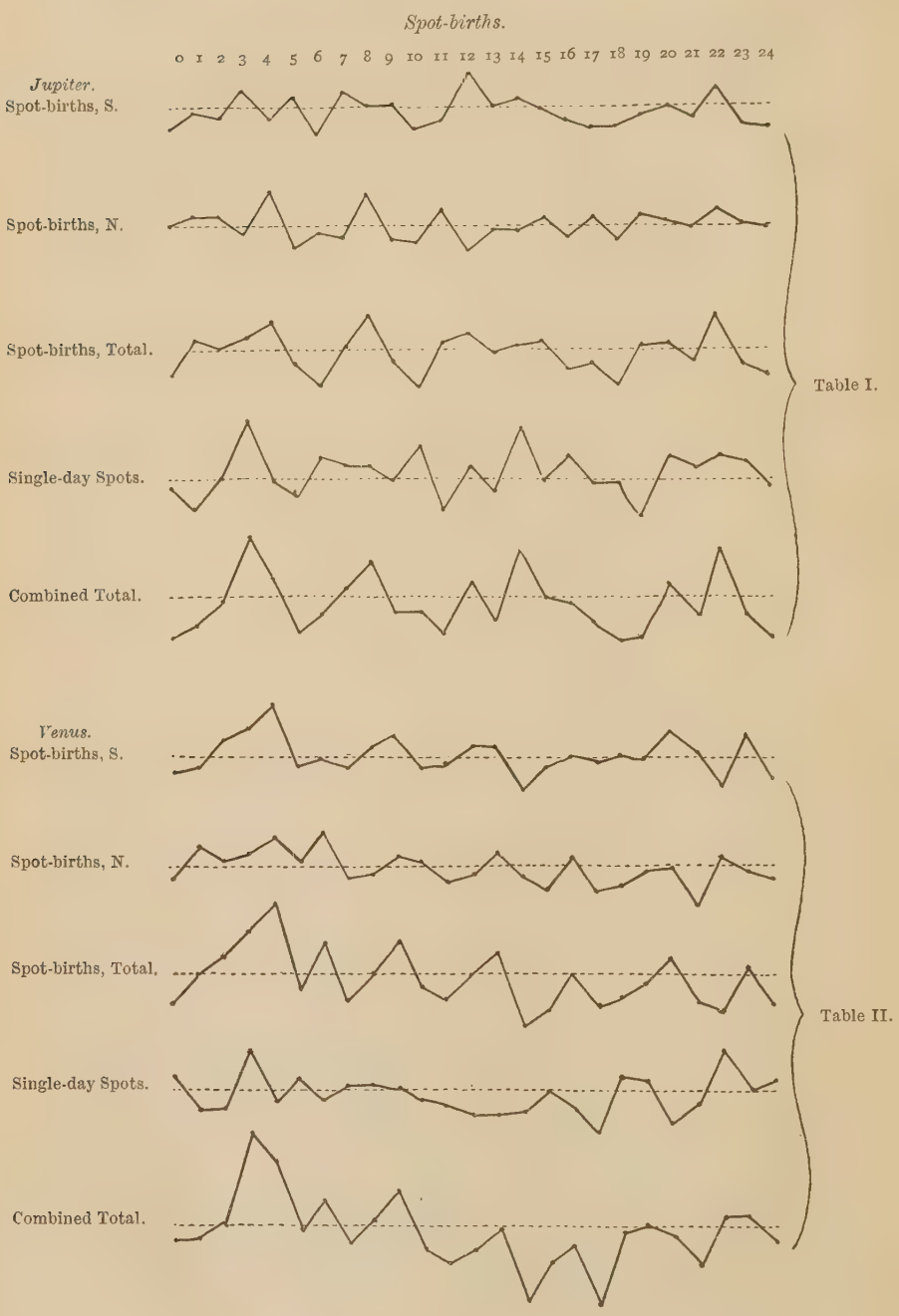
Spot-births.
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
Jupiter.
Spot-births, S.
Spot-births, N.
Spot-births, Total.
Single-day Spots.
Combined Total.
Table I.
Venus.
Spot-births, S.
Spot-births, N.
Spot-births, Total.
Single-day Spots.
Combined Total.
Table II.

FIG. 2.

TABLE II.

Spot-births and Single-day Spots observed within 50° of the central meridian, arranged in 24 hours of a Venus Day.

Hour.	Births.			Single-day Spots.			Combined Total.
	S.	N.	Total.	S.	N.	Total.	
1	40	48	88	15	27	42	130
2	52	42	94	23	19	42	136
3	58	47	105	37	30	67	172
4	64	52	116	25	20	45	161
5	40	42	82	29	24	53	135
6	46	54	100	30	17	47	147
7	41	37	78	27	24	51	129
8	50	38	88	28	23	51	139
9	56	44	100	27	23	50	150
10	40	41	81	23	24	47	128
11	42	35	77	21	23	44	121
12	50	38	88	26	14	40	128
13	50	45	95	27	13	40	135
14	30	37	67	20	21	41	108
15	40	31	71	22	27	49	120
16	44	43	87	28	14	42	129
17	43	31	74	16	18	34	108
18	46	32	78	28	27	55	133
19	45	38	83	22	31	53	136
20	54	39	93	17	22	39	132
21	49	28	77	19	26	45	122
22	31	42	73	37	30	67	140
23	51	39	90	28	23	51	141
24	38	37	75	28	27	55	130
Total	1100	960	2060	603	547	1150	3210
Average	46	40	86	25	23	48	134

Here we notice in the combined total two maxima, one at 3 and 4 h., and the other at 9 h.; the former is due to both sets of spots, the latter to the spots of more than one day's life. There is then fair agreement with two, and only two, of the four Jupiter maxima. If we compare the minima we find very little agreement, a moderate minimum at 11 h. in both being counter-balanced by several marked discordances.

§ 3. *Conclusions from spot-birth figures.*—In estimating the meaning of the above figures one has to remember that there are several disturbing causes at work which make it difficult to apply the ordinary canons of probability and to use the theory of random sampling. For instance, owing to the elliptic motions

of the Earth and the planets different lunes on the Sun (or hours of the planet-day) have unequal opportunities of showing spots to the Earth while within the limits assigned (50° from the observed central meridian). In the course of thirty years or more this inequality would have smoothed itself out were it not that certain epochs, those of sun-spot maximum, had so much more effect on the statistics than others. For one period of maximum, 1891–7, it was found that the maximum difference between the number of days during which separate lunes were in the central regions of the visible disc was about 2 per cent. of the time for either lune. The effect of this difference would be appreciable in the tables but would not be sufficient to account for irregularities of the size observed. A much more serious trouble arises from the fact that during a period of maximum activity extending over several years all the lunes do not appear equally frequently. The mere presence of a burst of activity over a limited time will almost certainly lead to inequalities in the statistics. The three or four maxima at present available seem too few to decide how far the inequalities found represent genuine phase effects of the type sought for and how far they are due to the accident of favourable opportunity. A careful study of the detailed figures underlying the tables in this paper, leaves the author unable to support the theory of a real planetary effect. Only in the case of the 3 h. maximum for Venus already referred to, does the inequality continue strongly throughout the series of observations. So far as Venus and Jupiter are concerned it seems impossible as yet to rule out chance as an explanation. While admitting that this explanation is more difficult when it has to be extended to cover the case of a third planet, the author feels that Dr. Schuster's coincidence argument * is incomplete: in so far as both Jupiter and Mercury have several marked maxima, the evidential value of the principal coincidence is much lessened by the presence of several marked discordances. Again if, as seems very probably the case with the periods of maximum activity, we have variations due to some outside factor superposed on the fluctuations of sampling, the theory of simple sampling cannot be applied satisfactorily. It may be applied, however, with some reservation as giving rather too favourable a verdict in favour of a real effect. In general practice a deviation from the mean of less than three times the standard deviation is regarded as ascribable to fluctuations of sampling.† In the totals of Tables I. and II., only one residual from the mean lies outside these limits. If we take it that the theoretical value of the deviation σ is given by

$$\sigma^2 = npq,$$

where n = number of spots considered,
p = probability that a spot be born in a given lune,
$q = 1-p$,

* *Loc. cit.*, p. 317.
† See Yule, *Theory of Statistics*, p. 262.

then the standard deviation for the Venus spot-births is 9·7. A residual of 30 in the figures for 3 h. does, according to the above criterion, call for explanation. Dr. Schuster suggests a planetary effect. To the present writer the evidence seems so uncertain that he prefers to suspend judgment until more figures are available.

§ 4. *Spot-deaths.*—Tables have been formed in a similar way to Tables I. and II. giving the Jupiter and Venus hour of all the final observations of spots last seen within 50° of the central meridian.

TABLE III.

Spot-deaths observed within 50° *of the central meridian, arranged in* 24 *hours of a Jupiter Day.*

Hour.	Deaths. S.	Deaths. N.	Total.	Single-day Spots.	Combined Total.
1	50	41	91	36	127
2	45	42	87	49	136
3	41	62	103	70	173
4	50	55	105	48	153
5	59	36	95	42	137
6	38	41	79	55	134
7	63	38	101	52	153
8	55	41	96	51	147
9	48	49	97	46	143
10	49	37	86	60	146
11	46	54	100	35	135
12	41	47	88	51	139
13	54	44	98	43	141
14	61	51	112	69	181
15	61	45	106	48	154
16	63	58	121	56	177
17	43	52	95	45	140
18	58	48	106	46	152
19	42	46	88	33	121
20	51	53	104	55	159
21	54	47	101	51	152
22	46	54	100	56	156
23	61	44	105	54	159
24	49	42	91	43	134
Total	1228	1127	2355	1194	3549
Average	51	47	98	50	148

TABLE IV.

Spot-deaths observed within 50° of the central meridian, arranged in 24 hours of a Venus Day.

Hour.	Deaths. S.	Deaths. N.	Total.	Single-day Spots.	Combined Total.
1	54	43	97	42	139
2	52	42	94	42	136
3	43	61	104	67	171
4	60	52	112	45	157
5	61	46	107	53	160
6	58	53	111	47	158
7	62	60	122	51	173
8	53	56	109	51	160
9	56	41	97	50	147
10	51	63	114	47	161
11	47	35	82	44	126
12	46	38	84	40	124
13	55	37	92	40	132
14	43	50	93	41	134
15	46	41	87	49	136
16	42	43	85	42	127
17	36	56	92	34	126
18	35	48	83	55	138
19	48	43	91	53	144
20	40	39	79	39	118
21	52	37	89	45	134
22	61	47	108	67	175
23	48	35	83	51	134
24	56	31	87	55	142
Total	1205	1097	2302	1150	3452
Average	50	46	96	48	144

The most striking fact revealed by a comparison of these tables with the earlier ones is that about 10 per cent. more deaths than births are recorded. This has already been noticed by Mrs. Maunder. Allowing for the greater number of spot-deaths recorded, it is of interest to examine whether similar phenomena are to be found for Jupiter and Venus. Taking 10 per cent. off the death figures, we find that for the central zones opposite both Jupiter and Venus the difference is in the opposite sense. The

natural conclusion is that the Earth effect is either accidental or of geometrical origin, depending on some rotational asymmetry of spot formations.

A comparison of the Jupiter and Venus spot-death tables shows

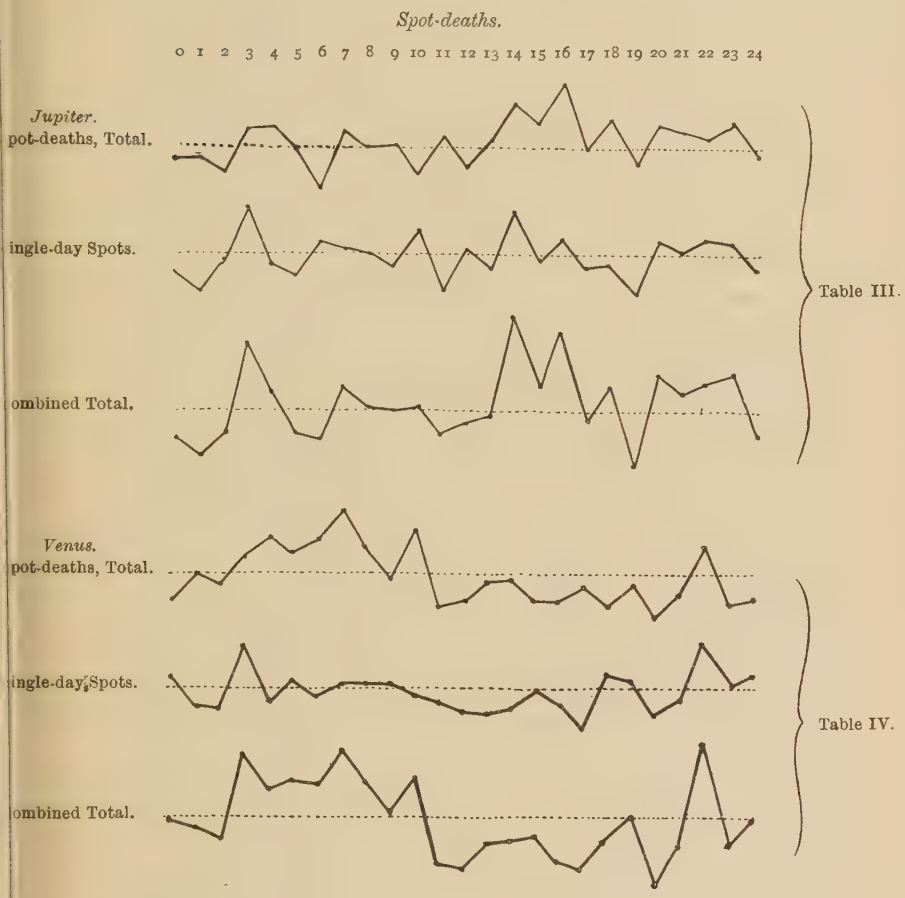

FIG. 3.

nowhere any marked agreement in phase; nor is there any marked opposition or agreement between the birth and death tables. If we divide the tables up into the four quadrants starting from 0^h, we find agreement in the distribution of births and deaths relative to Venus; but the results differ markedly from the case of Jupiter, where the maximum for births corresponds in position to the minimum for deaths.

TABLE V.

Distribution in Quadrants of Spot-births and Spot-deaths.

	Jupiter.		Venus.	
Hours.	Births.	Deaths.	Births.	Deaths.
0– 6	541	560	585	625
6–12	544	568	512	608
12–18	515	638	472	532
18–24	535	589	491	537

These figures show a less tendency for spots to be born and a greater tendency for spots to die at a place on the Sun during the six hours before Jupiter rises above the horizon, while births and deaths are both more frequent during the six hours after Venus crosses the meridian. In both cases a majority of births (1 per cent. of all births for Jupiter, 4·5 per cent. for Venus) takes place while the planet is above the horizon, contrary to the result given by Mrs. Maunder for the Earth; deaths are more frequent while Jupiter is invisible (by 3 per cent.), but while Venus is visible (by 1 per cent.). Agreement and disagreement are not so marked as to suggest the need of a physical explanation.

§ 5. *Spot Distribution.*—A further attempt has been made to trace connections between sun-spot phenomena and the planets by referring each entry of a spot in the Greenwich Tables to the appropriate hour of a planet-day. In this way it has been possible to deduce the tendencies of the various lunes to be spotted. No reference is here made to the spot-areas, which introduce their own special complications into every question, but the number of spots observed in each lune is alone considered. A long-lived spot is spread over a number of lunes, and in our final totals we have only to examine the distribution of the residuals from a mean number. In the following tables the limitation to a range of 50° on either side of the central meridian has been removed; otherwise the same explanations hold as in the corresponding tables (I. and II.).

TABLE VI.

Number of Spots observed in the course of a Jupiter Day.

Hour.	S.	Residual.	N.	Residual.	Total.	Residual.
1	729	−48	697	+ 1	1426	−47
2	740	−37	684	−12	1424	−49
3	779	+ 2	695	− 1	1474	+ 1
4	797	+20	664	−32	1461	−12
5	796	+19	664	−32	1460	−13
6	813	+36	653	−43	1466	− 7
7	829	+52	668	−28	1497	+24
8	819	+42	674	−22	1493	+20
9	818	+41	680	−16	1498	+25
10	791	+14	678	−18	1469	− 4

TABLE VI.—*continued.*

Hour.	S.	Residual.	N.	Residual.	Total.	Residual.
11	783	+ 6	660	−36	1443	−30
12	794	+17	690	− 6	1484	+11
13	809	+32	696	0	1505	+32
14	816	+39	716	+20	1532	+59
15	778	+ 1	726	+30	1504	+31
16	764	−13	727	+31	1491	+18
17	744	−33	730	+34	1474	+ 1
18	750	−27	699	+ 3	1449	−24
19	748	−29	713	+17	1461	−12
20	769	− 8	733	+37	1502	+29
21	755	−22	721	+25	1476	+ 3
22	744	−33	734	+38	1478	+ 5
23	729	−48	716	+20	1445	−28
24	749	−28	681	−15	1430	−43
Total	18642		16700		35342	
Average	777		696		1473	

TABLE VII.

Number of Spots observed in the course of a Venus Day.

Hour.	S.	Residual.	N.	Residual.	Total.	Residual.
1	716	+ 5	631	− 1	1347	+ 3
2	712	+ 1	632	0	1344	0
3	733	+22	672	+40	1405	+61
4	763	+52	665	+33	1428	+84
5	753	+42	663	+31	1416	+72
6	755	+44	673	+41	1428	+84
7	748	+37	664	+32	1412	+68
8	755	+44	646	+14	1401	+57
9	744	+33	637	+ 5	1381	+37
10	705	− 6	635	+ 3	1340	− 4
11	698	−13	624	− 8	1322	−22
12	721	+10	632	0	1353	+ 9
13	709	− 2	640	+ 8	1349	+ 5
14	660	−51	657	+25	1317	−27
15	655	−56	645	+13	1300	−44
16	659	−52	636	+ 4	1295	−49
17	654	−57	638	+ 6	1292	−52
18	681	−30	600	−32	1281	−63
19	683	−28	597	−35	1280	−64
20	678	−33	603	−29	1281	−63
21	708	− 3	597	−35	1305	−39
22	725	+14	598	−34	1323	−21
23	726	+15	598	−34	1324	−20
24	724	+13	597	−35	1321	−23
Total	17065		15180		32245	
Average	711		632		1344	

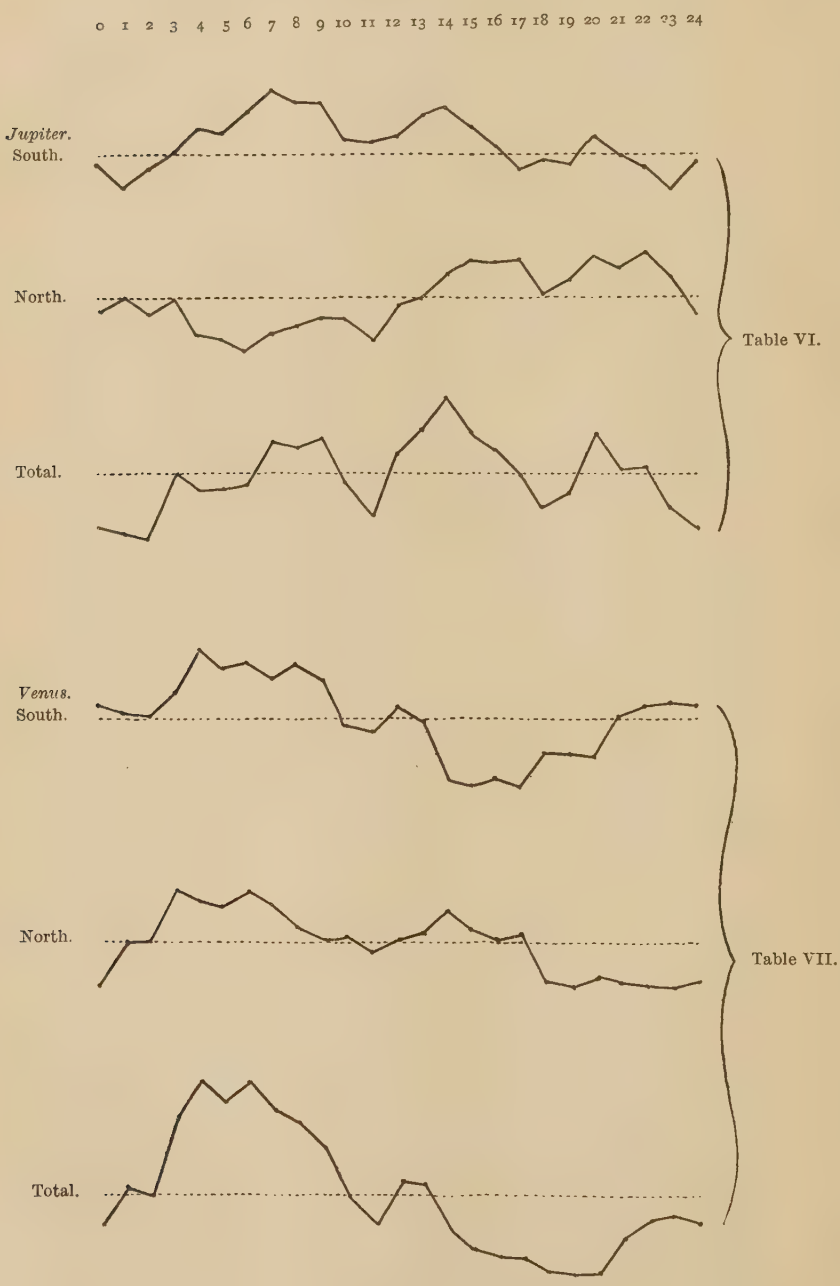
Spot Entries.
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
Jupiter.
South.
North.
Total.
Table VI.
Venus.
South.
North.
Total.
Table VII.

FIG. 4.

It will be noticed from Tables VI. and VII. that the residuals for the Venus southern and northern spots are in very fair accord, while the Jupiter southern and northern spots are in marked disagreement with each other.* It is not easy to suggest a physical theory which will cover the Venus and the Jupiter figures. The changes of phase in the four sets of figures point rather to an accidental distribution of the various maxima and minima arising from unequal incidence in time of the phenomena. It is curious, though not perhaps altogether unexpected, that the frequency distribution and the spot-birth tables should show much the same relations between the southern and northern spots for the two planets. The four-peaked frequency curve occurs for Jupiter in both cases, also the well-marked maximum and minimum for Venus. It is to be noticed here that an examination of Carrington's figures for four synodic revolutions of Venus (1854 February 28 to 1860 July 18) gives results very like those of Table VII.

TABLE VIII.

Number of Spots observed by Carrington referred to the hours of a Venus Day.

Hours.	S.		N.		Total.	
1	68	− 3	58	− 2	126	− 6
2	77	+ 6	69	+ 9	146	+14
3	89	+18	65	+ 5	154	+22
4	90	+19	70	+10	160	+28
5	88	+17	68	+ 8	156	+24
6	88	+17	62	+ 2	150	+18
7	88	+17	63	+ 3	151	+19
8	95	+24	58	− 2	153	+21
9	90	+19	57	− 3	147	+15
10	91	+20	68	+ 8	159	+27
11	84	+13	62	+ 2	146	+14
12	73	+ 2	65	+ 5	138	+ 6
13	71	0	69	+ 9	140	+ 8
14	72	+ 1	69	+ 9	141	+ 9
15	66	− 5	70	+10	136	+ 4
16	53	−18	66	+ 6	119	−13
17	54	−17	54	− 6	108	−24
18	53	−18	56	− 4	109	−23

* This is the only case where the correlation coefficient between the south and north columns of figures in any one of the above tables lies outside the range between $\pm 0{\cdot}5$; the value here $-0{\cdot}55$ for Jupiter corresponds to a value $+0{\cdot}34$ for Venus. No correlation coefficient between corresponding sets of figures for the two planets lies outside the range $+0{\cdot}5$ to $-0{\cdot}5$.

TABLE VIII.—*continued.*

Hours.	S.		N.		Total.	
19	46	−25	52	− 8	98	−34
20	52	−19	50	−10	102	−30
21	45	−26	49	−11	94	−38
22	51	−20	48	−12	99	−33
23	53	−18	47	−13	100	−32
24	72	+ 1	55	− 5	127	− 5
Total	1709		1450		3159	
Average	71		60		132	

It is certain that the differences in the southern spots in Table VIII. are largely to be traced to the fact that in the last and most active synodic revolution observations were made by Carrington with unequal regularity. Thus, on the average, the Sun was observed on twice as many days when the lunes 2–7 were visible as when the lunes 12–18 were visible. The figures of Table VIII. cannot therefore be adduced in strong support of any theory of planetary action; nor can the Carrington records be used in any way as a check on the spot-birth conclusions on account of the then inevitable irregularity of observation.

A few more facts of interest arising from Tables VI. and VII. may be pointed out. There are more spots on the side of the Sun invisible to Jupiter than on the visible side. To a less extent the opposite holds good for Venus. Mrs. Maunder has given good reason to believe that in this matter the Earth agrees with Jupiter. It should be noted that these results for both Venus and Jupiter are entirely due to the southern spots. The northern spots give opposite results.

To make another comparison where the Earth can be more directly introduced, consider the number of spot-entries for the eastern and western halves of the Sun's disc, visible from a planet. For the Earth we use the figures given by Mrs. Maunder (*loc. cit.*, p. 456).

TABLE IX.

Spots on the Visible Side of the Sun.

	E.	W.
Jupiter	8792	8711
Earth	8213	7508
Venus	7834	8368

As in several other cases the Earth and Jupiter agree, while Venus gives a different distribution. The chief point of interest is that a result of the same order of magnitude, though of opposite sign, is found for Venus as for the Earth. This weakens distinctly

the argument in support of a quasi-geometrical explanation of this inequality.

Still another comparison may be made which should throw light on any relation existing between sun-spots and the planets. The number of northern and southern spots recorded when any planet is north or south of the solar equator may give some measure of the repellent or attractive effect of a planet on sun spot formation. In the following table the periods used are complete periods between passages of a planet through the ascending node. For the Earth, individual spots only are recorded, while for the planets the figures for all spot-entries were extracted as in Tables VI. and VII.

TABLE X.

Planet.	South.		North.	
	Spots.		Spots.	
	S.	N.	S.	N.
Jupiter	8419	5621	5785	5931
Earth	1512	1254	1485	1329
Venus	6931	5750	7381	6212

Save for the southern spots referred to Venus, there is a striking regularity about these figures. Spots in either hemisphere are more frequent when the planet is on the same side of the Sun's equator as the hemisphere in question; the most striking example is given by the southern spots referred to Jupiter, 8419 entries when Jupiter is south of the Sun's equator as against 5785 when Jupiter is north. For the Earth these figures may be slightly discounted as due to foreshortening, but for Jupiter they are certainly suggestive of some real connection. If, however, we check this table by Carrington's figures we find the corresponding set of figures.

TABLE XI.

Jupiter.	South.		North.	
Spots.	S.	N.	S.	N.
No. of entries	1511	1481	1148	769

It is clear that not the position of Jupiter relative to the Sun's equator, but its position at the times of maximum, is the deciding factor in the figures of the above table. And that is the general conclusion to which the writer has been forced. The inequalities occurring in most sets of figures are traceable to the unequal incidence of the maxima in the various distributions and do not point directly to a physical connection.

§ 6. *Jupiter-Venus effects.*—One attempt has been made to trace cross-effects between two planets such as were suggested by Professor E. W. Brown (*M.N.R.A.S.*, vol. lx.). The spot entries were recorded in a double-entry table, classified by the hour of the

planet-day (Jupiter or Venus) and the angular distance subtended at the Sun by the two planets. Separate results were found for northern and southern spots and for Venus and Jupiter. No satisfactory conclusion could be derived from these tables, and it has seemed sufficient to print one table, that for Jupiter combining the southern and northern spots. The table is divided into two separate portions for reasons to be given later: each portion refers to a complete number of synodic revolutions of Venus relative to Jupiter, reckoned from conjunction. The headings of the columns represent (Heliocentric longitude of Venus—heliocentric longitude of Jupiter) expressed in hour angle.

TABLE XII.

Sun-spot Entries referred to a Jupiter Day, and the Relative Positions of Venus and Jupiter (1874–1905).

V.-J. / Hour.	0^h	3^h	6^h	9^h	12^h	15^h	18^h	21^h	24^h	Total.
1	173	150	129	156	145	153	186	157		1249
2	166	168	137	159	139	151	184	167		1271
3	183	181	134	153	141	158	177	171		1298
4	176	170	143	141	150	144	158	142		1224
5	190	178	140	133	148	142	157	145		1233
6	179	176	153	140	139	140	160	150		1237
7	184	176	149	145	153	138	160	166		1271
8	175	169	133	152	155	131	153	170		1238
9	177	166	136	150	147	141	159	160		1236
10	168	175	140	149	152	144	164	156		1248
11	160	156	147	150	154	150	163	159		1239
12	153	158	150	161	164	158	159	159		1262
13	159	164	147	176	160	165	168	165		1304
14	160	161	148	177	147	175	182	166		1316
15	149	153	144	181	153	160	179	167		1286
16	151	147	154	180	141	171	157	161		1262
17	158	142	154	187	141	176	149	146		1253
18	153	133	153	174	151	171	148	161		1244
19	150	143	143	171	151	175	149	161		1243
20	156	143	152	176	150	189	168	150		1284
21	158	140	157	160	149	175	165	158		1262
22	164	145	161	163	142	169	172	154		1270
23	173	141	158	170	130	171	178	153		1274
24	171	148	146	161	139	155	179	150		1249
Total	3986	3783	3508	3865	3541	3802	3974	3794		30253

TABLE XIII.

Sun-spot Entries as in Table XII. (1905–9).

V.-J. / Hour.	0^h	3^h	6^h	9^h	12^h	15^h	18^h	21^h	24^h	Total.	Table XII.	Combined Total XII. and XIII.
1	38	39	52	28	29	16	14	25		241	1249	1490
2	42	42	50	35	24	16	18	26		253	1271	1524
3	42	42	51	31	29	22	20	24		261	1298	1559
4	43	39	54	38	22	23	22	31		272	1224	1496
5	41	35	40	33	23	26	23	26		247	1233	1480
6	38	35	32	37	20	25	23	23		233	1237	1470
7	26	38	33	29	23	26	25	23		223	1271	1494
8	25	36	35	28	34	25	26	32		241	1238	1479
9	24	31	45	31	31	29	27	35		253	1236	1489
10	26	34	42	33	21	30	27	35		248	1248	1496
11	24	38	43	31	22	28	26	26		238	1239	1477
12	22	39	43	28	28	28	25	32		245	1262	1507
13	17	32	46	18	32	28	21	38		232	1304	1536
14	25	30	50	26	33	28	18	37		247	1316	1563
15	31	38	49	29	35	26	16	31		255	1286	1541
16	37	35	48	32	34	24	17	30		257	1262	1519
17	32	37	39	28	39	25	19	31		250	1253	1503
18	30	35	46	25	33	31	19	29		248	1244	1492
19	31	29	47	26	33	35	18	25		244	1243	1487
20	45	28	40	37	33	37	18	22		260	1284	1544
21	36	34	38	35	35	26	21	25		250	1262	1512
22	36	37	37	33	40	22	19	31		255	1270	1525
23	40	35	47	26	31	24	17	33		253	1274	1527
24	43	29	56	31	25	22	17	34		257	1249	1506
Total	794	847	1063	728	709	622	496	704		5963		
Table XII.	3986	3783	3508	3865	3541	3802	3974	3794			30253	
Combined Total XII. and XIII.	4780	4630	4571	4593	4250	4424	4470	4498				36216

The two tables were obtained separately as explained in § 1. They have been printed separately to show by an example how the addition of a few years' figures alters the tabular results. Thus, in Table XII. there is a strongly defined minimum for the case where the difference in hour angle of the two planets lies between 6^h and 9^h. The same column in Table XIII. shows a very pronounced maximum. In the combined total this column occupies a middle position. It is an interchange of position such as this which seems to throw doubt on conclusions drawn from the present limited series of figures.

We note that there are more spots recorded when Jupiter and Venus are within six hours of one another in hour angle to a solar observer than when their longitudes differ by a greater amount. Here we may compare the figures with the corresponding Earth-Jupiter and Earth-Venus figures.

TABLE XIV.

Distribution of Spots relative to Pairs of Planets.

Pairs of Planets.	Difference of Heliocentric Longitude.	
	$<6^h$.	$>6^h$.
Venus-Earth	16202	16043
Venus-Jupiter	18378	17838
Earth-Jupiter	17503	17839

We note as before that Venus behaves differently from the other two planets, but the figures are not decisive at all of any physical effect.

§ 7. *Summary of Conclusions.*—The foregoing notes are mainly negative in character, suggesting that the case for planetary effects on sun-spot phenomena is "Not proven." The hope with which the investigation was originally undertaken has not, however, been abandoned. The writer's view is still that the dynamical effects of planets upon what may be an atmosphere of very uncertain stability should be manifested in some solar phenomena. Sufficient data are not yet accumulated to smooth out the effects of the far more efficient forces which cause the great periodic outbursts of solar activity, and no certain conclusions seem as yet to be within the reach of the investigator.

It remains for the writer to express his indebtedness to Mr. C. Dolman for the efficient way in which he has carried through much of the necessarily heavy work of computation.

Cambridge Observatory:
1911 *October.*

Empirical Short Period Terms in the Moon's Mean Longitude.

By Frank E. Ross.

(*Communicated by P. H. Cowell, D.Sc., F.R.S.*)

In the *Monthly Notices*, R.A.S., vol. lxix. p. 166, Newcomb tabulates the final outstanding residuals, in the sense O − C, of the Moon's mean longitude. His comparison is based upon pure theory, with the addition of the long period empirical term

$$\delta_1\lambda = 12''\cdot95 \sin [100^\circ\cdot6 + 131^\circ\cdot00\ (T - 1800\cdot0)].$$
(T = centuries).

The residuals show the presence of other terms of comparatively short period, which is well marked from about 1850 to 1908. Their analysis discloses the following terms, which are to be taken as additive to Newcomb's mean longitude:—

$$\delta_2\lambda = -0''\cdot18 + 2''\cdot9 \sin [350^\circ\cdot6 + 6^\circ\cdot316\ (t - 1900\cdot0)]$$
$$+ 0''\cdot8 \sin [313^\circ + 15^\circ\cdot65\ (t - 1900\cdot0)].$$
(t = Julian years.)

These terms are of periods fifty-seven and twenty-three years respectively. In deriving them, small weight was given to Newcomb's residuals before 1840.

It should be stated that before making the analysis, Newcomb's residuals were corrected by the amount—

$$+0''\cdot50 \sin \Omega. \quad (\Omega = \text{long. of Moon's node.})$$

This is equivalent to adopting a smaller value of the constant "ellipticity" factor than that found by Hill and used by Newcomb.

The following table shows the resulting values of the residuals, v, after applying $\delta_2\lambda$. Comparison with Newcomb's values (minor residuals) in *Monthly Notices* shows that the earlier observations are apparently better represented without short period empirical terms. This, however, is irreconcilable with the much more accurate observations of the latter part of the last century. The question whether the Moon's mean longitude can be represented by any mathematical formulæ is for the future to decide.

[Table.

Outstanding Residuals in the Moon's Mean Longitude.

Year.	*v.*	Year.	*v.*	Year.	*v.*	Year.	*v.*
	″		″		″		″
1681·0	−3·0	1849·5	−0·3	1869·5	+1·3	1890·5	−0·2
1710·0	+3·2	50·5	+0·5	70·5	+1·0	91·5	+0·1
27·0	+0·5	51·5	+0·2	71·5	−0·5	92·5	+0·5
37·0	−1·0	52·5	+0·1	72·5	−0·5	93·5	+0·3
47·0	−3·6	53·5	−0·4	73·5	−0·3	94·5	−0·1
55·0	−1·0	54·5	−0·1	74·5	−0·4	95·5	+0·4
71·0	+3·2	55·5	+0·2	75·5	−0·5	96·5	+1·0
84·7	+3·0	56·5	−0·4	76·5	−0·3	97·5	−0·2
92·0	−1·7	57·5	−0·7	78·0	0·0	98·5	−0·1
1801·5	−2·9	58·5	+0·4	79·5	+1·3	99·5	−0·4
09·0	−1·7	59·5	+0·4	80·5	+0·3	1900·5	+0·6
13·0	−2·1	60·5	+0·2	81·5	+0·1	01·5	0·0
21·0	+1·4	61·5	−0·5	82·5	+0·4	02·5	0·0
22·5	+2·4	62·5	+0·2	83·5	−0·3	03·5	−0·1
29·5	+2·2	63·5	−0·5	84·5	−0·1	04·5	+0·1
33·5	+1·9	64·5	−0·2	85·5	−0·2	05·5	+0·2
38·5	+0·7	65·5	−0·1	86·5	−0·4	06·5	−0·3
43·0	+0·7	66·5	+0·6	87·5	+0·1	07·5	+0·1
46·5	+0·6	67·5	−0·4	88·5	−0·6	08·2	+0·1
48·5	+1·4	68·5	+0·1	89·5	−0·4		

Washington, D.C.

Note on the Two Main Types of Cometary Development and their Variation with the Solar Distance. By J. H. Reynolds.

At the March meeting of the Society I drew attention to the two principal types of cometary development.

The first type is that in which the tail is merely a continuation of the parabolic envelopes of the head, formed from matter projected from the nucleus towards the Sun; this may be termed the cylindrical type, and is associated with large comets near perihelion, such as 1910*a*. In the other type the tail is formed from streamers radiating from a point in the nucleus directly away from the Sun; this may be termed the conical type, and of this Morehouse's Comet was a conspicuous example.

From an examination of the Helwân photographs of Halley's Comet, it was evident that both types were represented in the series, the cylindrical type being developed when the comet was near perihelion at a distance of 0·5 to 0·6R, and the conical type coming into prominence when the solar distance was greater than 0·8R.

As it seemed, therefore, probable that the type of tail development depended on the solar distance, a preliminary examination

of the photographic records of other comets was made by Mr. Knox Shaw and myself. Unfortunately these records are very incomplete, and usually only a few isolated photographs of each comet are available. At the same time the results were fairly consistent, and the following provisional classification was adopted, to which there were no marked exceptions :—

Solar distance under 0·6R—Cylindrical type.
„ 0·6 to 0·8R—Intermediate, combining both the cylindrical and the conical types.
„ over 0·8R —Conical type.

The only series of photographs in the library of the Society comparable with the Helwân series of Halley's Comet were the Greenwich photographs of Morehouse's Comet. As, however, the perihelion distance of this comet was over 0·9R, and the latest photograph in the library was taken over a month before perihelion passage, this series has no bearing on the point at issue, except that the conical form of tail was consistently shown.

Recently two comets have appeared which go far towards putting this provisional classification on a firm basis.

Beljawsky's Comet (1911 *d*), photographed at Helwân at a solar distance of between 0·3R and 0·4R, showed distinctly the cylindrical type. Photographs of Brooks' Comet (1911 *c*), taken by Mr. Longbottom at Chester, showed a well-developed conical tail, while the solar distance was 1·0R and over. When near perihelion, at about 0·5R, it appears visually to have developed the cylindrical type, and this is confirmed by photographs taken at that time.

Sextant Observations of Comets Brooks and Beljawsky.
By Captain Thomas, *S.S. Ovid.*

[The following notes have been communicated to the Society by the Meteorological Office.]

Comet Brooks.

September 26, 1911.

At 8 p.m. A.T.S., in lat. 36° 32′ N., long. 16° 19′ E., G.M.T. $26^{d}\ 6^{h}\ 44^{m}$, observed a comet in a position 10° 17′ from Benetnasch Ursa Major and 42° 14′ from Polaris.

September 27.

At 7.21 p.m. (G.M.T. $27^{d}\ 6^{h}\ 24^{m}$), in lat. 36° 54′ N., long. 12° 0′ E., the above comet was in a position 9° 17′ from Benetnasch, 43° 28′ from Polaris, and 28° 50′ from Arcturus.

Comet Beljawsky.

October 2, 1911, at 4 a.m.

In lat. 36° 15′ N., long. 6° 30′ W., observed a comet in a position 17° 28′ east of Regulus, and 11° 45′ north of Venus. Greenwich mean time $1^{d}\ 16^{h}\ 15^{m}$.

Royal Observatory, Edinburgh. Note on the Spectrum of Comet Brooks (1911 *c*). (Plate 1.)

(*Communicated by the Astronomer Royal for Scotland.*)

Photographs of the spectrum of Comet Brooks (1911 *c*) have been obtained on fourteen days between September 1 and October 31, with a prismatic camera, having a lens of Iceland spar, 2 inches aperture and 29 inches focal length, and a quartz prism 60° angle, giving a dispersion between H_β(4861·5) and H_ι(3770·8) of 12·15 mm.

On October 27 and 31 exposures were made with a portrait doublet of 3 inches aperture and 12 inches focal length, with a glass prism of 60° angle, the resulting spectra being the most intense and showing the tail well. The corresponding dispersion was 18·1 mm. between H_β and H_ι; Sirius giving the comparison spectrum.

The plates used were Ilford "Monarch," backed, except on September 8 (Barnet Super-Speed Ortho) and September 21 (Panchromatic), the exposures ranging from 201 minutes to 50 minutes.

As long an exposure as could be obtained was always given, owing to the faintness of the comet, and to its not being very active photographically, and the resulting negatives differ greatly in the amount of detail according to the exposure obtained.

The cameras were rotated so as to have the edge of the prism approximately parallel to the comet's tail, but it was not until the end of October that a definite tail could be traced in the spectra.

With the spar camera no trace could be found of duplicity in the knots, such as is recorded in the spectrum of Comet Morehouse (1908 *c*), the brightest knot λ 467 being, in particular, noted as symmetrical and round. The knot λ 388 was very much flatter, and on one or two negatives gave the impression of a slight shading towards the violet.

With the other camera the knot λ 467 is distinctly double, the fainter image being on the violet side. None of the other knots showed signs of duplicity.

As early as September 8 a faint continuous spectrum could be traced, and this phenomenon was easily visible as the brightness of the comet increased. This comet differs in that respect from Comet Morehouse, in which the continuous spectrum was absent.

The photographs were measured in the Hilger measuring machine—1 revolution of micrometer screw = 1 mm. Except for the two brighter knots λ 388 and λ 457, it was impossible to set the micrometer wire with any degree of accuracy on the knots themselves owing to the magnification (15). In the case of several of them any magnification greater than that given by a hand lens

made them disappear altogether. To remedy this the following device was adopted:—Retouching medium was applied to the film and a fine pencil-mark made at the point desired. The plate was then measured in the direct and reversed positions, and all marks cleaned off with alcohol. After several days the same process was repeated, each marking being made quite independently of the other.

For the two brighter knots the edges were generally well enough defined for measurement, as well as the centre, but the edges of the others were not sharp enough to warrant measurement except at their estimated centre.

A specimen may be given to show the accordance obtained in the measures derived from the marks put upon the images which were too faint for direct measurement. The images in question are the last two of the spectrum in the extreme ultra violet. The two numbers given for each date represent the two independent markings as described above, each separate number being the mean of measures with the plate first direct and then reversed.

	r.	*r.*		*r.*	*r.*
Sept. 8	19·024	26·440	Oct. 6	18·850	
	18·988	26·388		18·835	
20	18·756		10	18·654	26·370
	18·619			18·727	26·269
24	18·930	26·751	26	19·021	
	19·037	26·639		18·909	
27	18·931	26·414	27	18·519	
	18·949	26·441		18·884	
30	18·683	26·094	Means	18·833	26·404
	18·658	26·235			
30	18·785				
	18·910				

In these parts of the spectrum 1^r is equivalent respectively to 48 A.U. and to 36 A.U.

The photographs were all made and measured by Mr. J. Storey.

The determination of the wave-lengths was made as follows. Upon several of the plates taken with the quartz prism, spectra of Vega were also photographed. The best of these was that of September 27 and it was used, to the exclusion of the others. From it, with H_β as origin, measures of the rest of the hydrogen series were taken, and also of the middle of the comet's image at λ 3872. On comparing the measures, expressed in revolutions of the micrometer screw, with the wave-lengths of the hydrogen

lines, it was found that the following formula would represent them all, with errors of very few units in the last place:

$$r = 8{\cdot}709 - 1{\cdot}366\Lambda + 0{\cdot}557\Lambda^2 - {\cdot}00167\Lambda^3,$$

where $\Lambda = \frac{1}{100}$ (wave-length − 4000),

r = revolutions of the screw.

This formula was found by charting the values of r, Λ, and noticing that they nearly followed a parabola with axis parallel to $\Lambda = 0$. The equation of such a parabola can be found, practically exactly, by direct and easy steps, and the small discordances which it leaves point to the value of the fourth term. A small table was then constructed showing the values of r corresponding to values of Λ from -10 upwards. The hydrogen series ends between $\Lambda = -2$ and $\Lambda = -3$; hence for $\Lambda = -9$ and -10, where the last observed image lies, the formula is much less certain than could be wished, but it may be mentioned that it agrees exactly with the result of free-hand continuation of the charted values made before the formula was obtained. The cometary measures on each of the plates were made relatively to that at λ 3872 in terms of revolutions of the screw, and the corresponding wave-lengths read from the table. This applies to the photographs taken with the quartz prism; for those taken with the glass prism the circumstances did not permit a well-placed comparison spectrum of a star to be printed, but one of Sirius was obtained upon the plate, which was measured and compared with the measures of Vega with the quartz prism. The comparison showed that, measuring again from the line H_β as origin, corresponding portions of the spectrum were roughly one-third larger in the former case than in the latter, the ratio increasing towards the violet, or, exactly, that revolutions (r) measured upon the glass-prism plates could be reduced to equivalent measures upon the other plates by multiplying them by the factor

$$\cdot 7637 - {\cdot}00570\, r + {\cdot}000032\, r^2.$$

With the glass prism, the image at λ 3872 appears double; the middle of this double was taken to coincide with the middle of the corresponding images from the quartz prism, and the wave-lengths of the remaining lines deduced, by first converting the relative revolutions of the screw into their equivalent upon the spar plates, and then reading off wave-lengths from the table already employed.

The following are the resultant measures, with estimates of the relative intensities of the condensations, and a few notes added:—

SPECTRUM OF COMET BROOKS (1911 c).

Taken with 3-inch portrait lens and 60° glass prism, 1911 Oct. 27. Exposure 30m. Enlarged 3·2 times from original negative.

SEPT. 30, $10^h\ 12^m - 11^h\ 57^m$.

OCT. 27, $17^h\ 15^m - 17^h\ 45^m$.

COMET 1911 c (BROOKS).

Taken with 4-inch portrait lens of 33-inches focal length.

The photographs illustrate the change in the type of tail which was noticeable when the Comet reappeared as a morning star.

Quartz Prism.				Glass Prism.			
Wave-length.	Intensity.	Number of Plates where visible.	Notes.	Wave-length.	Intensity.	Number of Plates where visible.	Notes.
5386	2	1					
5024	2	7					
				4841	1	2	Tail only.
4712			Beginning.	4712			Beginning.
4672	12	14	Query tail.	4681	15	2	Head and short tail.
4634			End.	4658			End.
4559	3	3	Faint tail; perhaps head also.	4545	4	2	Strong tail; possibly head.
4353	3	10	No tail.	4358	3	2	No tail.
4251	4	1	Tail only; strongest.	4254	5	2	No head; second strongest tail.
4200	3	8	No tail.	4196	4	2	Head only.
				4114	1	2	Tail just visible.
4022	2	12	Second strongest tail.				
				4006	5	2	Strongest tail.
3889			Beginning.				
3872	20	14	Head; short tail.	3876	20	2	Head strong, double.
3858			End.	3869	14		Perhaps tail.
				3775	1	1	Very faint tail only.
3677	1	1	Very faint tail.				
3580	3	3	Head; fairly strong tail.				
3351	4	10	Head; probably tail also.				
3057	3	5					

A comparison of this spectrum with that of Comet Morehouse (1908 *c*) shows at once that they are almost identical except for the circumstance of the presence of a continuous spectrum in Comet Brooks, and a few other details. Taking the comparison from the paper by MM. de la Baume Pluvinel and Baldet in *Astrophysical Journal*, 1911 September, the line 5386 may possibly be the same as the unidentified 5369 given by these authors, the extrapolation making in our case the measure somewhat uncertain. 5024 corresponds with their 5021, which they attribute to CO, though it is among the cases where their numbers are a little discordant from Fowler's. The image of the tail at 4841 agrees with their 4846, which is also apparently a discrepant member of the CO series, but yet a case in which they feel able to insist

upon their number. Next is a hydrocarbon band. M. Bosler, in *Comptes Rendus*, 1911 October 23, remarks that this band extends into the tail about 30′: "alors que le spectre des hydrocarbures et celui du cyanogène sont pour ainsi dire limités à la tête." The images of the tail at 4559, 4545 are due to CO. The images of the head at 4353, 4358 seem due to carbon, which was represented in Comet Morehouse by a faint image at 4372. The measures 4251, 4254 belonging to the tail only are due to CO. 4200, 4196, belonging to the head only, as well as the group with middle at 3872, are due to CN. In the spectrum given by the glass prism, this shows neatly double. The images of the tail at 4022, 4006 represent another image of the CO series. The image of the tail at 3775 seems to represent the CO pair that in Comet Morehouse was found at 3783 and 3803. The image at 3677 is very faint, but seems to represent the CO pair 3687, 3701. There remain the images of the tail at 4114, and the two images of the head shown by the quartz prism deep in the ultra violet, measured at 3351 and 3057. These last are extrapolated so far beyond the point where the hydrogen series ceases that it is impossible to insist closely upon the numbers. Still it is evident, from plotting the measured portion upon a chart, that they cannot be far wrong, unless the character of the dispersion by the quartz suffers a great and rapid change in this portion of the ultra violet. As far as appears, they do not seem to have been observed before.

Royal Observatory, Edinburgh. Astrographic Measures of Double Stars (Zone −40°). By R. W. Wrigley, B.A., F.R.S.E.

(*Communicated by the Astronomer Royal for Scotland.*)

The following is a list of the double stars measured on the plates comprising zone −40° of the Astrographic Catalogue, and therefore includes stars between the parallels of declination 39° and 41° south. The plates were taken at the Perth Observatory (Western Australia), mainly in 1903–4, and the measures have been made at the Royal Observatory, Edinburgh. The distance and position angle for each pair of stars have been obtained from the ordinary rectangular co-ordinates by a method similar to that described by Mr. H. C. Plummer (*Monthly Notices*, lxix. 2). If (x, y), $(x+\delta x, y+\delta y)$ represent the standard co-ordinates of any pair of stars referred to a corner of the plate as origin, then according to the notation used for this zone—

$$\text{Distance between the stars} = \left\{ (\delta x)^2 + (\delta y)^2 \right\}^{\frac{1}{2}} \times 300''$$

$$\text{Position angle in degrees} = \tan^{-1} \frac{\delta x}{\delta y} + 1^\circ (x - 14) \tan (\text{dec.}) \div 12$$

As might have been expected, very few pairs closer than 3″

could be measured. For the 6-minute exposure the average diameter of images of tenth magnitude stars was fully 3″, and for the ninth magnitude was 5″·5, so that the components of any measurable pair whose separation was less than 4″ were in general necessarily faint. However, in several cases where the 6-minute images of the two stars were hopelessly coalesced, excellent differential measures could be obtained from the 20-sec. images, especially when the components were of nearly equal magnitude. For bright stars exceeding the 6th magnitude, measures of well-known pairs were frequently impossible, a notable example being θ Eridani (mags. 3·3 and 4·8, distance 8″), the six images formed by this pair being completely united and indistinguishable. The large majority of the pairs in the list are of separation less than 10″, with about one hundred less than 5″, but a few wider pairs measured by Sir J. Herschel have been included. The comparative futility of frequent observations of such wide pairs is shown by a comparison of the measures of these stars in 1835 with those given below.

A few examples of such a comparison will be found at the end of the present paper. The measures of Herschel are given in juxtaposition with those from the astrographic plates, the position angles of the former being corrected for precession. The separation of the pairs ranges from 16″ to 70″. In each case the position angles for the two measures are practically identical, and the distances show no appreciable change, especially as Herschel's distance measures during his reflector sweeps were necessarily somewhat rough.

In the following table :—

Columns I. and II. give the Right Ascension and South Declination for 1900·0.

Columns III. and IV. give the Distance and Position Angle in seconds of arc and degrees respectively.

Column V. gives the difference in magnitude between the two components. This has been obtained from the measured diameters of the 6^m images, according to the Greenwich formula, magnitude $= a + b\sqrt{\text{diameter}}$, a and b being constants.

Column VI. gives the date of the exposure of the plate.

Columns VII. and VIII. give the number in the Cape Photographic Durchmusterung of the star with which the pair in question has been identified, and the magnitude of that star. When no identification could be made in the C.P.D., the Cordoba Durchmusterung was employed, and in such cases the numbers are enclosed in brackets. In general the magnitudes, as derived from the diameters on the astrographic plates, showed close agreement with the C.P.D., but in a few cases the C.D. magnitude appeared more correct, and has therefore been entered in Column VIII. These cases are distinguished by brackets.

Column IX. gives the name of the pair whenever it has been identified, but no attempt has been made to do this with any degree of thoroughness. Numbers following the hour of R.A.

refer to the Reference Catalogue of Southern Double Stars (*Annals of Cape Observatory*, vol. ii. part 2), and, as usual, *h* denotes Sir J. Herschel, and λ the Lowell Observatory. While a considerable number of old pairs may have escaped identification as such, there must remain in the list below many whose positions and measures are now published for the first time. An asterisk in the last column denotes that the stars appeared on two plates, the mean of the measures being adopted. The reliance to be placed upon double star measures derived from photographs is to some degree an unknown quantity, as it is known that neighbouring images on a plate may displace one another's positions owing to distortion of the film. Where the separation exceeds 4″ the measures seem to be of nearly the same order of accuracy as visual observations for stars fainter than the ninth magnitude, and for large distances the photographic method seems the more consistent, but for bright stars closer than 10″ the advantage lies the other way. The following are two examples of double stars of separation about 10″, the astrographic measures being compared with those made by skilled micrometrical observers.

Ref. Cat. $10^h{\cdot}78$. *h.* 4373. *Combined Mag.* 8·8.

Distance.	Position Angle.	Epoch.	Observer.
25″±	226°·1	1835·2	*h.*
12·66	337·0	1879·4	Hargrave.
10·44	339·1	1882·3	,,
6·01	...	1892·5	Tebbutt.
10·8	344·0	1900·4	Innes.
11·0	344·5	1902·5	,,
10·74	345·6	1903·4	Astrographic.
10·50	346·2	1903·3	,,

Ref. Cat. $18^h{\cdot}7$. *h.* 5023. *Combined Mag.* 8·0.

Distance.	Position Angle.	Epoch.	Observer.
9″·76	275°·8	1836·5	*h.*
8·99	276·5	1837·5	,,
10·42	276·3	1879·3	C.G.A.
5·85	277·5	1880·7	Hargrave.
	277·3	1900·6	Lunt.
9·14	276·2	1900·6	,,
8·97	275·1	1903·6	Astrographic.

	R.A.	South Declination.	Dist.	Position Angle.	Diff. of Mags.	Epoch.	No. in C.P.D.	Combined Mag.	Name.
	h m	° ′	″	°		1900+	°		
1	0 4·6	39 19	3·42	229·0	1·2	4·928	(39·22)	10	
2	10·2	40 39	13·80	3·0	1·5	,,	40· {13 14}	7·5	
3	19·8	40 21	5·88	39·9	1·4	,,	40·28	9·6	
4	27·2	40 30	8·49	344·4	0·7	,,	40·44	(9·7)	
5	43·0	39 47	11·82	241·6	0·7	,,	39·53	9·6	
6	44·4	39 41	8·64	290·6	0·5	,,	39·56	8·7	*h.* 3399.
7	0 53·4	39 17	5·61	130·4	=	,,	(39·247)	10	
8	1 17·4	40 34	4·83	158·6	1·1	4·852	(40·321)	9·7	
9	39·9	39 2	8·79	8·8	0·6	,,	(39·526)	9·9	
10	42·4	40 27	8·94	269·6	1·4	,,	40·155	7·8	*h.* 3465.
11	42·0	40 56	8·16	79·5	0·7	,,	(41·485)	9·9	
12	1 51·9	39 22	2·76	202·1	0·2	,,	39·159	(9·4)	
13	2 21·5	40 23	9·75	325·0	1·4	4·975	(40·614)	9·9	
14	33·3	41 2	2·91	248·2	0·4	5·016	41·234	9·2	
15	35·7	40 24	3·63	322·8	0·7	,,	48·222	9·0	2^h 24. Innes 53.
16	41·4	40 59	9·18	346·4	0·8	,,	41·260	9·4	
17	2 52·7	39 51	3·45	184·2	0·5	,,	39·249	8·0	2^h 47. Cordoba 7.
18	3 11·1	39 2	3·81	100·0	0·6	5·019	39·284	(9·6)	
19	24·9	40 30	6·03	353·7	1·0	4·977	(40·939)	9·7	
20	36·2	40 41	8·01	323·9	0·7	5·019	40·343	6·6	3^h 37. Dunlop 15.
21	40·6	40 58	4·26	349·3	3·3	,,	41·382	6·6	3^h 39. *h.* 3589.
22	42·2	40 29	3·42	208·5	0·5	,,	40·357	8·7	Cape 38.
23	43·0	40 59	15·42	139·5	2·1	,,	41·388	8·8	*h.* 3593.
24	3 53·1	40 12	3·90	147·1	0·6	5·011	40·385	7·4	3^h 55. *h.* 3611.
25	4 18·7	39 21	6·00	251·2	0·5	,,	39·433	9·5	
26	46·7	39 21	4·11	56·4	0·7	4·134	39·520	8·6	4^h 52. Cordoba 11.
27	50·0	40 44	7·86	18·7	0·2	,,	(40·1631)	10	
28	4 58·7	39 44	12·51	196·2	1·6	4·096	39·567	9·6	*h.* 3717.
29	5 3·7	39 47	24·17	322·8	0·4	,,	39· {582 583}	9·4 9·8	*h.* 3725.
30	5·6	39 36	5·10	284·8	1·7	,,	39·588	9·5	
31	7·8	39 36	6·60	257·8	=	,,	(39·1823)	10	
32	15·1	39 56	8·80	164·1	0·2	,,		9·9	*
33	15·9	39 9	6·06	86·7	1·5	,,	39·636	9·4	
34	25·9	39 33	4·59	230·8	1·2	,,		10	
35	27·1	39 57	7·14	151·2	0·3	,,	39·688	10·1	
36	28·6	40 27	6·24	276·0	0·9	,,	40·741	8·8	*h.* 3769.
37	30·3	40 17	3·69	243·4	0·6	,,		10	
38	5 31·3	39 1	5·34	102·7	1·1	4·134	39·704	9·2	

	R.A.	South Declination.	Dist.	Position Angle.	Diff. of Mags.	Epoch.	No. in C.P.D.	Combined Mag.	Name.
	h m	° ′	″	°		1900+	°		
39	5 38·7	39 28	5·64	248·4	0·2	4·096	(39·2130)	10	
40	38·7	40 11	2·40	360·2	0·3	,,		10·4	
41	42·2	40 21	7·23	353·7	1·5	,,	40·809	9·6	
42	45·6	39 12	3·54	127·7	1·0	,,		10	
43	47·4	39 28	14·52	116·6	=	,,	(39·2221)	10	*h.* 3806.
44	48·8	40 16	6·54 16·08	117·3 118·9	1·0 1·3	,,	40·843	9·9	A – B. A – C.
45	5 56·8	40 30	3·72	104·4	0·8	4·131		9·8	
46	6 1·8	40 12	2·76	101·6	0·8	,,	40·906	9·9	
47	2·4	39 34	10·29	129·7	0·5	,,	39·842	10·0	
48	4·3	40 3	3·93	177·1	1·3	,,	40·919	10·0	
49	8·4	40 23	5·31	34·2	1·1	,,	40·938	8·8	6^h 13? Cordoba G.C. [7459.
50	16·5	39 27	40·26	52·7	0·7	,,	39·904 39·906	7·8 8·5	*h.* 3849.
51	20·0	39 28	7·86	296·6	0·4	,,	39·923	9·8	
52	22·6	40 55	8·88	238·5	1·0	,,	40·1011	7·5	*h.* 3860.
53	21·9	39 26	10·08	208·2	0·4	,,	39·929	9·4	
54	23·5	38 56	2·82	143·6	1·3	,,	38·885	9·4	
55	35·7	40 30	28·26	160·6	2·2	,,	40·1100	8·1	*h.* 3881.
56	37·3	40 0	5·16	215·7	=	,,	39·1036	9·8	
57	38·0	40 15	16·89	107·2	4·1	,,	40·1111	6·5	*h.* 5443.
58	42·6	39 49	4·71	192·5	1·3	,,	39·1081	9·4	
59	6 50·6	39 32	3·54	210·6	0·3	,,	39·1145	10·3	
60	7 20·9	40 39	4·71	116·4	0·1	,,	40·1456	9·8	
61	22·2	40 43	6·15	54·0	0·4	,,	40·1467	9·8	
62	23·1	39 25	3·90	105·1	0·3	,,	39·1410	10·1	
63	23·6	39 7	4·41	213·3	0·4	,,	39·1409	9·6	
64	24·4	40 49	3·99	283·9	1·5	4·216	40·1490	(8·6)	
65	24·8	39 38	7·02	41·7	0·6	4·174	39·1432	9·5	
66	29·2	40 50	3·69	22·2	0·4	4·216	40·1537	9·8	
67	29·7	39 8	5·82	148·5	0·5	,,	39·1478	9·8	
68	35·9	38 58	6·27	209·2	0·8	,,		10·3	
69	38·7	39 21	4·02	27·0	0·8	,,	39·1588	9·8	
70	40·3	39 38	7·02	7·9	0·8	,,	39·1618	7·6	
71	44·3	40 23	4·53	153·8	0·3	,,		10·4	
72	47·1	39 30	4·71	97·4	0·6	,,	39·1762	9·6	
73	49·7	40 35	3·42	209·1	0·6	,,	40·1874	10·4	
74	51·1	40 16	3·75	195·2	0·6	,,	40·1897	9·4	
75	56·2	40 12	6·66	152·3	0·3	,,		10·4	
76	7 57·6	39 56	5·88	206·0	1·0	,,	39·1951	9·0	

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	h m	° ′	″	°		1900+			
77	7 58·8	39 15	3·45	55·8	0·3	4·216		10·5	
78	7 59·3	41 2	26·85	346·9	4 0	,,	40°2039	5·7	*h.* 4030.
79	8 1·1	38 56	6·48	40·3	0·6	,,	38°1910	9·8	
80	2·2	39 19	4·74	27·6	0·6	,,	39°2060	10·1	
81	2·5	39 8	4·50	267·7	0·1	,,		10·5	
82	4·6	40 54	9·16	97·6	=	,,	40°2141	10·2	*
83	7·2	39 31	6·00	161·9	0·8	,,	39°2160	10·4	
84	7·6	39 2	8·70	125·0	0·5	,,	38°2030	9·9	
85	8·9	39 31	3·30	245·5	=	,,		10·7	
86	8·9	40 7	6·21	29·9	=	,,	40°2217	9·4	
87	10·3	40 16	6·36	18·8	1·8	,,	40°2257	9·2	
88	10·5	40 2	49·02	340·8	4·0	,,	39° 2218 / 2219	9·2 / 7·0	*h.* 4062.
89	10·6	40 27	6·15	269·9	0·4	,,	(40°4003)	9·3	
90	12·2	40 46	5·97	232·7	0·5	,,	40°2297	9·4	
91	14·8	40 25	5·49	59·5	0·7	,,	40°2336	8·6	
92	18·2	39 25	3·12	264·8	0·2	,,		10·6	
93	18·6	40 40	13·35	345·5	3·5	,,	40°2395	7·7	*h.* 4087 (A + B, C).
94	20·5	40 1	9·39	287·1	0·2	,,	(39°4292)	9·6	
95	22·3	39 14	8·61	207·6	0·9	,,	39°2390	9·0	*h.* 4092.
96	24·4	39 46	14·82	277·3	2·1	,,	39°2435	8·6	*h.* 4098.
97	24·6	39 0	11·01	213·2	0·1	,,	38°2339	9·9	
98	25·4	40 10	7·32	31·2	0·9	,,	40°2536	9·2	*
99	28·6	40 47	6·15	247·7	2·2	,,	40°2587	9·0	
100	28·9	40 56	8·43	150·8	0·2	,,	40°2600	10·0	
101	32·3	39 38	6·36	239·2	0·4	,,	39°2612	9·2	
102	32·7	40 39	2·82	217·7	0·5	,,	40°2680	9·2	*h.* 4114.
103	34·4	39 14	11·52	87·8	2·0	,,	39°2670	9·4	
104	36·8	40 3	6·36	83·1	1·0	,,	39°2753	9·6	
105	37·0	40 43	17·04	321·9	3·7	,,	40°2749	8·0	
106	37·2	40 48	2·37	198·2	0·1	,,	40°2750	9·6	
107	39·3	39 32	4·53	226·2	1·0	,,	39°2790	9·6	
108	40·5	40 13	4·02	342·5	0·2	,,	40°2811	9·2	
109	41·0	40 55	10·05	325·4	1·0	,,	40°2823	9·4	
110	41·1	39 7	4·17	64·2	=	,,		10·5	
111	41·2	40 28	4·41	1·7	0·2	,,		10·1	
112	43·1	39 8	8·04	320·0	2·5	,,	39°2860	7·6	*h.* 4138.
113	43·7	40 1	2·25	194·7	=	,,	39°2874	10·4	
114	8 43·4	40 10	5·97	175·1	1·2	,,	40°2867	10·0	

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	h m	° ′	″	°		1900 +	°		
115	8 44·2	39 5	7·47	56·4	1·1	4·216	38·2704	10·0	
116	46·2	40 1	2·25	20·0	0·2	4·263	39·2917	8·5	
117	46·9	39 51	6·78	342·4	0·3	,,	39·2930	9·4	
118	48·2	40 27	3·21	232·9	0·4	,,		9·9	
119	48·4	39 34	2·31	218·5	0·2	,,		10·6	
120	49·1	41 3	16·44	197·9	1·3	,,	40·2940	(8·0)	
121	52·2	41 3	8·67	291·0	1·7	,,	40·2990	9·6	
122	52·4	40 38	4·74	342·3	0·3	,,	40·2995	10·1	
123	8 53·3	39 13	5·37	156·4	0·9	,,	39·3054	10·0	
124	9 2·9	39 31	9·63	63·8	0·3	3·326	39·3203	9·4	
125	10·8	39 20	3·39	102·7	0·3	,,	39·3339	10·2	
126	13·3	40 16	5·73	159·5	0·7	,,	40·3353	10·0	
127	42·0	40 42	6·87	110·1	1·4	,,	40·3867	9·2	
128	43·3	39 25	9·63	234·7	1·2	,,	39·3897	9·4	
129	9 54·6	39 53	4·95	177·9	0·3	,,	39·4082	10·2	
130	10 0·7	40 13	8·37	270·5	1·1	,,	40·4179	9·5	
131	13·8	39 44	5·67	321·5	1·3	,,	39·4324	8·2	
132	19·7	41 0	5·37	236·1	0·3	,,	40·4424	10·4	
133	26·0	39 42	5·40	56·2	1·1	,,	39·4466	10·4	
134	31·8	39 16	10·47	87·9	0·2	,,	39·4538	10·0	
135	39·6	39 34	5·52	247·7	0·6	3·424	39·4617	8·6	*h.* 4349.
136	43·0	40 1	10·71	57·8	0·1	3·375	39·4663	9·8	
137*	44·4	40 55	10·62	345·9	0·3	,,	40·4750	8·8	10h 78. *h.* 4373.
138	45·0	41 4	6·09	176·0	0·2	,,	40·4757	8·6	
139	10 58·5	40 31	8·31	151·8	0·8	3·329	40·4916	8·6	
140	11 2·5	39 3	5·22	114·7	2·0	,,	38·4543	9·0	
141	2·5	40 55	18·72	347·8	3·0	,,	40·4967	8·0	*h.* 4408.
142	4·7	40 28	3·39	301·2	1·0	,,	40·4989	10·0	
143	20·4	39 11	8·25	16·9	1·7	3·424	39·5068	8·8	
144	37·0	40 2	3·72	150·8	0·6	3·435	39·5222	10·2	
145	38·9	40 47	5·22	180·9	1·1	,,	40·5345	10·0	
146	39·4	39 17	5·37	300·0	2·4	,,	39·5243	8·6	
147	53·2	39 5	4·71	327·5	1·4	3·329	38·4944	8·0	
148	56·8	40 27	7·35	292·0	0·9	3·435	40·5493	9·9	
149	11 59·7	39 26	2·19	215·7	1·4	,,	39·5380	9·4	
150	12 4·8	39 44	3·97	207·8	1·6	3·430	39·5421	9·4	*
151	11·0	40 40	8·19	55·6	0·2	3·424	40·5605	10·2	
152	19·5	40 50	10·05	208·6	1·2	3·435	40·5688	7·5	12h 35. *h.* 4518.
153	12 27·5	39 34	4·05	184·7	0·5	,,	39·5589	9·8	

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	h m	° ′	″	°		1900+	°		
154	12 31·9	40 16	8·22	178·9	2·1	3·435	40·5823	8·2	Russell 203.
155	41·7	40 32	8·46	249·1	=	,,	40·5908	10·4	
156	41·8	39 9	4·17	248·7	0·1	,,	39·5727	10·5	
157	48·2	39 4	6·60	349·8	0·8	3·438	38·5340	10·0	
158	48·1	39 45	8·22	231·6	0·7	,,	39·5758	10·2	
159	51·2	39 43	23·37	163·9	2·5	,,	39·5781	8·6	Z.C. 12h 2919.
160	53·1	40 26	20·52	346·5	2·6	,,	40·5975	8·1	λ_1 166.
161	12 54·2	39 43	4·56	228·3	0·3	,,	39·5808	10·5	
162	13 9·5	39 7	6·09	65·4	1·3	3·435	38·5459	9·6	
163	19·4	40 37	3·24	174·2	0·3	,,			
164	20·5	39 27	25·17 7·98	270·7 170·4	0·6 2·2	,, ,,	39·5979 39·5978	9·6 9·1	A.B. A.C.
165	21·4	39 39	44·40	139·3	3·9	,,	39·5984	7·3	*h.* 4588.
166	24·8	40 25	2·81	181·7	0·1	,,	40·6188	9·8	*
167	33·3	39 9	11·34	283·3	0·2	,,	39·6032 39·8382	9·2 8·8	*h.* 4601.
168	38·1	39 5	6·15	79·3	2·2	3·438	38·5574	9·5	
169	48·0	40 5	8·37	277·3	0·7	,,	39·6111	9·4	
170	48·6	40 18	5·52	64·5	0·1	,,		10·2	
171	51·7	39 29	3·69	31·5	0·5	,,	39·6131	9·0	*h.* 4636.
172	56·1	39 32	4·23	277·5	0·8	,,	39·6168	10·0	
173	58·2	41 0	4·74	156·0	=	,,	40·6429	9·6	
174	13 58·7	40 59	9·33	26·9	2·6	,,	40·6433	8·2	
175	14 1·1	39 22	3·84	352·2	0·7	,,	39·6206	9·2	
176	3·6	39 27	7·08	83·1	=	,,	39·6219	10·5	
177	4·4	38 55	4·98	145·8	=	,,	38·5741	8·3	
178	9·3	40 30	2·82	154·9	1·0	,,	40·6503	10·1	
179	10·9	39 7	5·73	119·6	0·2	,,	39·6256	9·7	
180	17·6	40 34	5·46	322·8	0·1	3·471	40·6561	10·6	
181	21·9	39 19	2·40	171·1	0·3	,,	39·6316	10·6	
182	39·3	39 37	8·52	3·3	2·2	,,	39·6418	8·7	
183	14 44·9	40 3	3·69	229·9	0·4	,,	(39·9279)	9·7	
184	15 3·8	40 38	4·98	74·2	0·5	,,	40·6840	7·6	15h 5. Cordoba [38].
185	51·0	38 58	6·36	39·9	1·2	3·474	38·6305	8·6	
186	15 54·2	40 9	8·67	157·3	3·4	,,	40·7168	6·4	15h 105. Cordoba [42].
187	16 13·8	39 11	9·72	298·2	2·9	3·641	39·6934	6·3	16h 28. Innes 91.
188	18·4	40 24	4·62	242·8	1·8	,,	40·7293	9·0	λ_1 274.
189	24·4	40 1	5·32	254·7	1·0	,,	39·6953	9·2	*
190	47·4	40 7	5·97	92·7	0·4	,,	40·7575	9·8	
191	16 47·8	40 43	9·06	176·4	2·5	,,	40·7587	8·4	

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	h m	° ′	″	°		1900+	°		
192	16 47·9	40 33	6·27	102·8	0·4	3·641	40·7594	10·4	
193	49·3	40 33	9·09	260·6	1·4	,,	40·7629	9·2	
194	49·8	39 41	7·92	87·8	0·2	,,	39·7185	(9·8)	
195	50·0	40 21	8·67 15·69	254·6 239·1	1·6 1·7	,, ,,	40·7650	7·0	A–B A–C Cordoba.
196	50·3	40 21	9·63	157·7	1·2	,,	40·7660	(9·8)	
197	52·0	39 25	4·20	7·8	1·4	,,	39·7197	8·8	
198	55·0	39 36	3·36	121·5	0·2	,,		10·6	
199	16 55·1	39 35	5·39	185·4	=	,,	39·7229	10·4	*
200	17 7·5	39 39	14·10 17·13	333·3 209·6	1·0 1·9	,, ,,	39·7296	7·9	17^h 16. *h.* 4926.
201	8·1	39 59	5·67	90·3	0·2	,,		10·6	
202	8·4	39 32	4·80	241·5	0·2	,,	39·7303	9·7	17^h 18. Innes 229.
203	12·1	39 40	4·74	34·4	0·7	,,	39·7325	8·6	Innes 409.
204	13·5	39 48	7·11	182·1	0·7	,,	39·7337	9·7	
205	15·7	40 6	8·19	19·9	0·2	,,	(40·11376)	10	
206	16·3	39 40	4·68	150·6	0·2	,,	39·7352	9·0	
207	21·3	40 28	5·82 19·86	50·0 179·5	0·5 0·6	,, ,,	40·7841 ·7842	9·7 9·5	B–C. A–C.
208	21·6	40 36	8·49	52·6	0·1	,,	40·7856	9·7	
209	26·5	39 49	3·42	151·7	0·7	,,	39·7397	9·3	
210	28·9	40 56	4·02	122·7	0·4	,,	40·7913	9·2	
211	30·1	40 47	10·89	140·1	0·9	,,	40·7926	9·4	
212	30·3	40 15	12·66	340·8	1·8	,,	40·7929	8·0	
213	30·5	40 34	3·09	14·5	0·2	,,	40·7933	9·3	
214	38·2	39 55	3·51	40·1	0·3	,,	39·7566	9·8	
215	41·0	40 6	5·43	262·0	0·5	,,	40·8101	8·8	17^h 91. Innes 108.
216	43·2	40 3	32·04	37·2	5·6	,,	40·8138	5·9	17^h 96. ι_2 Scorpii.
217	49·9	39 55	3·00	120·7	0·3	,,	39·7761	8·1	17^h 109. Cordoba [50].
218	50·4	39 25	9·81	97·8	1·1	,,	39·7773	9·4	
219	51·4	40 8	10·17	222·0	1·1	,,	40·8277	9·2	
220	54·0	40 7	6·99	69·4	1·3	,,	40·8327	9·0	
221	55·2	39 39	9·63	175·4	=	,,	39·7854	10·0	
222	57·3	40 19	8·31	30·9	0·3	,,	40·8381	10·0	
223	17 59·0	39 28	2·82	58·1	=	,,	39·7931	9·6	
224	18 2·0	40 10	5·85	355 3	0·3	,,	40·8474	9·6	
225	3·7	40 34	45·45	100·5	1·0	,,	40·8487	9·8	*h.* 5025.
226	3·8	40 27	8·97	275·1	0·1	,,	40·8488	7·8	18^h 7. *h.* 5023.
227	4·6	40 36	4·23	187·4	=	,,	(40·12241)	10	
228	18 5·2	39 22	13·99	147·2	0·7	3·686	39·7993 39·7994	8·2 8·8	*β.* 759. (AB–C). *h.* 5028. *

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	h m	° ′	″	°		1900+	°		
229	18 9·7	40 36	7·05	201·2	2·7	3·686	40·8516	8·1	
230	10·4	39 58	1·47	84·2	=	,,	(39·12408)	9·5	
231	13·9	40 24	8·88	64·1	0·3	,,	(40·12403)	10	
232	14·3	40 27	5·61	208·1	2·4	,,	40·8538	8·4	
233	15·1	40 49	11·19	8·3	0·8	,,	40·8544	(8·7)	
234	23·7	40 22	6·63	206·3	0·3	3·641	(40·12559)	9·7	
235	29·5	39 2	2·61	169·3	0·6	,,	39·8130	10·2	
236	43·1	39 28	14·19	219·4	2·7	3·731	39·8190	8·4	
237	45·8	39 57	5·64	306·7	0·5	,,	39·8203	10·2	
238	49·4	40 48	6·12	134·9	0·3	,,	40·8697	10·2	
239	52·3	39 40	16·26	245·4	4·2	,,	39·8235	7·2	*h.* 5074
240	18 53·3	39 53	5·07	205·0	0·2	,,		10·6	
241	19 2·3	40 16	5·37	229·1	0·5	,,	(40·13070)	10	
242	9·3	40 32	2·34	172·0	=	,,	40·8868	10·8	
243	20·3	40 5	10·86	65·7	0·3	,,	40·8988	8·7	*h.* 5115.
244	20·4	40 14	5·85	119·2	0·4	,,	40·8990	9·8	λ_1 374.
245	35·7	39 46	6·39	268·5	0·5	3·739		10·5	
246	37·0	39 9	9·54	125·8	2·8	,,	39·8514	[8·5]	λ_1 392.
247	41·0	40 13	4·11	220·5	0·1	,,	40·9105	(9·8)	
248	52·9	40 46	24·96	33·1	0·1	,,	40·9153 40·9154	10·0 10·2	*h.* 5159.
249	19 54·1	39 54	11·31	115·2	0·2	,,	39·8586	(9·6)	
250	20 0·0	40 48	16·95	329·7	0·4	4·756	40·9194	9·2	
251	5·6	40 14	9·18	306·3	0·5	,,		10·4	*
252	10·7	40 22	3·84	95·7	=	,,		10·7	
253	11·1	40 30	10·74	117·9	0·4	,,	40·9268	7·2	20h 20. Dunlop 230.
254	11·8	40 19	12·39	30·8	0·6	,,	40·9271	(9·6)	
255	16·8	41 3	5·13	33·1	0·7	,,	(41·14003)	9·9	
256	24·8	39 27	9·96	115·7	1·0	,,	39·8715	9·8	* *h.* 5203.
257	25·4	39 45	20·91	228·8	1·4	,,	39·8718	8·8	*
258	28·2	40 19	8·43	238·5	0·2	,,	40·9320	10·4	
259	28·7	40 38	8·19	5·5	2·5	,,	40·9321	8·4	λ_1 421.
260	33·8	40 33	6·72	57·2	0·3	,,		10·6	
261	27·1	40 54	4·65	220·3	0·9	,,	40·9315	7·5	20h 49. Jacob [11].
262	36·7	40 6	6·96	211·3	1·0	,,	40·9348	10·4	
263	51·2	39 31	7·35	33·0	=	,,	39·8812	10	
264	20 52·9	39 25	6·39	79·8	0·9	,,	(39·14051)	10	
265	21 0·1	40 36	6·39	64·8	0·3	,,	40·9457	9·6	
266	21 1·0	39 6	6·63	307·8	0·1	,,	39·8869	10·9	

	R.A.	South Declination.	Dist.	Position Angle.	Diff. of Mags.	Epoch.	No. in C.P.D.	Combined Mag.	Name.
	h m	° ′	″	°		1900+	°		
267	21 7·7	40 1	8·67	281·4	0·5	4·756	40·9495	(9·8)	
268	7·9	39 0	10·11	182·7	0·7	,,	39·8905	7·6	*h.* 5253.
269	11·3	39 39	5·61	206·9	=	,,	39·8919	10·4	λ_1 442.
270	14·9	40 37	4·77	52·2	0·8	,,	40·9521	(9·9)	
271	21·5	39 26	4·35	122·7	=	4·772	39·8949	8·4	
272	22·2	39 32	8·19	85·7	1·4	,,	39·8950	9·6	
273	23·1	39 16	7·86	25·6	2·4	,,	39·8956	8·0	21^h 34. Innes 133.
274	29·7	38 56	8·58	201·6	0·9	,,	(39·14324)	9·6	
275	33·5	39 43	3·24	62·4	0·5	,,	39·9002	9·8	
276	36·8	39 53	3·27	39·2	1·2	,,	40·9589	9·7	
277	36·2	38 56	4·14	62·0	0·6	,,	39·9007	10·2	
278	48·2	40 25	32·85	240·3	0·2	,,	40·9617 40·9618	9·0 9·0	*h.* 5299.
279	21 49·4	40 21	4·56	33·3	0·4	,,	(40·14548)	10	
280	22 18·2	40 14	5·40	275·1	=	,,	40·9688	9·3	*h.* 5330.
281	21·3	40 58	6·30	185·3	0·4	,,	41·9791	8·3	
282	22·8	39 38	27·09	115·9	4·4	,,	39·9127	6·7	λ_1 473.
283	29·8	39 14	4·80	172·9	2·0	4·776	39·9147	7·4	22^h 35. *h.* 5344.
284	35·6	39 52	8·16	234·0	2·5	,,	40·9721	8·8	*
285	51·2	39 28	11·79	348·8	1·2	,,	39·9186	[9·4]	
286	22 51·4	40 55	7·53	251·3	0·3	,,	41·9852	[9·6]	
287	23 13·5	40 57	4·23	39·2	0·4	,,	41·9911	8·7	
288	31·0	40 52	5·01	281·9	0·1	4·824	41·9948	10·1	
289	23 32·2	40 54	4·11	250·6	=	,,	41·9952	10·0	

Comparison of Herschel's measures of wide pairs with those above.

R.A.	S. Decl.	Distance.	Position Angle.	Epoch.	Name.
h m	°	″	°		
5 3·7	39·47	30 ±	321·3	1836·93	*h.* 3725.
		24·17	322·8	1904·41	
6 16·5	39·27	39·43	52·7	1837·99	*h.* 3849.
		40·26	52·7	1904·13	
6 38·0	40·15	17·25	105·0	1836·01	*h.* 5443.
		16·89	107·2	1904·13	
6 25·9	40·19	71·00	110·7	1838·09	B. 1254+1256.
		68·28	112·6	1904·13	
6 35·7	40·30	30 ±	157·8	1836·93	*h.* 3881.
		28·26	160·6	1904·13	
18 3·7	40·34	47·93	100·0	1836·51	*h.* 5025.
		45·45	100·5	1903·64	
19 52·9	40·46	20 ±	33·5	1834·63	*h.* 5159.
		24·96	33·1	1903·74	
21 48·2	40·25	30 ±	241·3	1835·71	*h.* 5299.
		32·85	240·3	1904·77	

Nouvelles étoiles doubles. 6ème Série.

Par Robert Jonckheere.

Disposant les étoiles par ordre de séparation angulaire, nous avons les chiffres suivants :

Distance.	Nombre de couples.	Pourcentage.
0″·0 à 2″	157	26·2
2·1 à 3	141	23·5
3·1 à 4	149	24·8
4·1 à 5	132	22·0
5·1 à +	21	3·5
	600	100·0

J.	Catalogue.	Æ. 1920. h m s	Dec. ° ′	Date. 1911	P. °	D. ″	Grandeurs.		n.
516	− 0° 3354	17 41 19	− 0 31	·565	240·8	3·47	9·0	9·8	1
				·565	235·0	3·55	9·1	9·8	1
517	+ 0° 3822	17 53 37	+ 0 15	·565	283·8	3·90	9·0	11·0	1
				·565	286·0	4·05	9·0	11·0	1
518	Anon.	18 1 29	+ 0 51	·554	143·7	4·31	9·4	10·7	2
				·554	142·0	4·51	9·3	10·6	2
519	− 14° 5014	17 48	− 14 39	·647	150·4	4·36	9·0	9·9	1
				·647	150·1	4·83	9·0	10·0	1
520	− 14° 5052	22 21	− 14 31	·647	292·6	5·01	9·1	9·1	1
				·647	293·8	5·14	9·1	9·1	1
521	+ 4° 3754	24 24	+ 4 4	·540	303·3	4·99	9·2	9·4	1
				·540	303·9	5·18	9·2	9·4	1
522	− 5° 4669	25 4	− 5 3	·653	218·9	4·77	9·3	9·9	1
				·653	217·8	4·89	9·3	10·1	1
523	+ 9° 3790	33 57	+ 9 42	·565	21·8	3·83	9·3	9·6	1
				·565	21·8	3·92	9·2	9·5	1
524	− 12° 5130	35 4	− 12 54	·647	308·6	3·80	8·8	9·9	1
				·647	312·2	3·64	8·9	10·0	1
525	Anon.	35 28	+ 29 32	·554	65·2	2·86	9·4	9·4	1
				·554	66·4	3·10	9·3	9·3	1
526	+ 8° 3802	37 18	+ 8 35	·565	236·9	2·80	9·0	9·7	1
				·565	238·3	2·71	9·1	9·6	1
527	+ 4° 3850	39 26	+ 4 57	·540	38·5	3·40	9·0	9·7	1
				·540	38·6	3·49	9·1	9·6	1
528	− 3° 4365	18 39 58	− 3 18	·653	13·5	4·40	9·0	9·0	1
				·653	14·1	4·62	9·1	9·1	1

J.	Catalogue.	Æ. 1920. h m s	Dec. ° ′	Date. 1911	P. °	D. ″	Grandeurs.		n.
529	−15° 5112	18 45 53	− 15 6	·647	349·4	5·00	9·4	12·0	1
				·647	351·9	5·51	9·3	11·7	1
530	Lpz. II. 8848	46 34	+ 8 44	·604	182·9	3·53	8·7	9·9	2
				·604	183·7	3·33	8·8	10·0	2
531	− 2° 4781	50 37	− 2 35	·653	158·6	4·55	9·5	9·8	1
				·653	156·9	4·33	9·4	9·7	1
532	Anon.	51 11	+ 12 54	·565	9·5	2·89	9·7	11·5	1
533	+ 8° 3948	18 58 11	+ 8 39	·543	5·8	10·55	9·0	10·0	1
	BC			·543	77·8	3·07	10·0	10·0	1
534	+12° 3783	19 0 5	+ 12 26	·644	3·9	4·59	9·3	9·4	1
				·644	4·8	4·46	9·4	9·4	1
535	− 8° 4895	9 15	− 8 33	·647	313·9	4·36	8·9	10·0	1
				·647	319·1	4·05	9·0	10·1	1
536	Anon.	10 51	− 8 32	·647	262·5	4·79	9·4	14·0	1
537	Anon.	11 43	+ 14 39	·565	196·5	4·15	9·5	9·7	1
				·565	194·2	4·13	9·6	9·8	1
538	Anon.	11 45	+ 22 36	·554	49·6	4·20	9·5	9·5	1
				·554	48·0	4·20	9·5	9·5	1
539	Anon.	18 30	− 1 28	·653	3·3	3·26	9·4	11·0	1
				·653	7·0	3·44	9·3	10·5	1
540	+ 9° 4125	26 54	+ 9 18	·551	122·6	3·12	9·3	9·3	1
				·551	124·7	3·43	9·3	9·4	1
541	Anon.	35 16	+ 15 16	·540	213·0	3·64	9·2	10·0	1
				·540	214·9	3·48	9·2	10·0	1
542	Anon.	49 41	+ 24 28	·591	181·9	3·43	9·5	11·5	2
				·568	186·0	3·78	9·5	11·5	1
543	Anon.	51 26	+ 5 26	·590	9·2	4·03	9·6	9·6	1
				·590	9·1	4·03	9·6	9·7	1
544	− 1° 3882	19 57 35	− 1 33	·776	354·9	3·78	9·2	10·0	1
				·776	352·5	3·83	9·2	9·9	1
545	+ 9° 4382	20 0 52	+ 9 35	·612	356·5	4·58	9·5	11·5	1
				·612	355·1	4·63	9·4	11·2	1
546	+ 7° 4361	1 52	+ 7 34	·609	294·4	2·21	9·0	9·5	1
				·609	290·0	2·33	9·0	9·4	1
547	Anon.	4 29	+ 6 15	·642	86·1	4·47	9·2	12·0	1
				·642	89·1	4·20	9·3	12·0	1
548	Anon.	7 53	− 0 46	·628	161·3	3·93	9 5	12·0	1
549	+10° 4210	9 9	+ 10 14	·642	298·2	4·72	9·0	12·0	1
				·642	300·5	4·67	9·1	12·0	1
550	Anon.	20 9 29	+ 4 25	·759	133·4	4·94	9·2	9·6	1
				·759	134·5	4·90	9·3	9·7	1

J.	Catalogue.	Æ. 1920.	Dec.	Date.	P.	D.	Grandeurs.		n.
		h m s	° ′	1911	°	″			
551	Anon.	20 9 57	− 1 7	·642	65·0	0·43	9·6	9·8	1
				·642	56·3	0·45	9·6	9·6	1
552	+ 8° 4386	11 5	+ 8 20	·601	274·6	3·90	9·2	9·5	1
				·601	275·1	3·81	9·3	9·5	1
553	Anon.	14 26	+ 15 39	·631	15·7	3·67	9·4	10·5	1
				·631	15·0	3·78	9·5	10·8	1
554	Anon.	15 27	+ 21 15	·546	210·5	3·94	9·5	10·0	1
				·546	214·3	4·11	9·4	10·2	1
555	Anon.	16 17	+ 1 2	·779	272·1	1·87	9·4	9·7	1
				·779	274·2	1·91	9·5	9·8	1
556	Anon.	19 43	+ 4 49	·759	187·2	2·67	9·4	9·4	1
				·706	186·0	2·78	9·3	9·3	2
557	+24° 4113	20 32	+ 24 49	·619	287·4	2·28	9·2	9·4	2
				·625	287·1	2·20	9·2	9·3	1
558	Anon.	21 30	+ 26 3	·642	184·4	4·60	9·5	12·5	1
559	Anon.	23 16	+ 9 32	·601	216·9	2·57	9·6	9·6	1
				·601	214·9	2·70	9·6	9·6	1
560	Anon.	23 24	+ 0 49	·590	185·5	3·83	9·4	9·8	1
				·590	184·9	4·02	9·4	9·7	1
561	Anon.	26 52	+ 1 55	·590	180·0	4·3.	9·3	10·0	1
				·590	181·5	4·52	9 4	10·0	1
562	+12° 4360	27 20	+ 12 31	·601	149·0	1·95	9·0	9·9	1
				·601	144·6	2·21	9·0	10·2	1
563	Anon.	27 25	+ 16 34	·601	192·3	4·97	10·5	12·0	1 BC
	Roe. 14			·601	262·3	7·37	9·6	10·5	1 AB
564	+21° 4245	27 41	+ 22 10	·628	306·1	1·63	8·9	11·6	3
				·773	303·4	2·04	9·0	12·0	1
565	+28° 3797	31 17	+ 28 57	·644	192·6	3·78	9·3	11·0	1
				·644	195·2	3·62	9·4	11·0	1
566	+15° 4204	31 25	+ 15 16	·642	118·7	4·52	8·7	10·0	1
				·642	123·5	4·30	8·9	10·0	1
567	− 1° 4011	32 38	− 1 19	·628	318·0	1·76	9·5	11·5	1
568	Anon.	33 33	+ 31 39	·644	351·9	4·83	9·5	9·7	1
				·644	350·1	4·90	9·5	9·8	1
569	Anon.	34 37	+ 1 39	·590	318·7	3·05	9·0	9·7	1
				·590	316·8	3·28	9·0	9·6	1
570	+25° 4311	35 38	+ 26 7	·642	95·3	4·92	9·3	12·0	1
				·642	96·8	4·75	9·3	11·8	1
571	Anon.	20 38 19	+ 7 7	·779	253·1	3·75	9·4	11·0	1
				·779	255·1	3·90	9·4	10·7	1

J.	Catalogue.	Æ. 1920. h m s	Dec. ° ′	Date. 1911	P. °	D. ″	Grandeurs.		n.
572	+ 4° 4565	20 48 41	+ 5 9	·614	143·5	3·64	8·9	10·0	2
				·614	141·9	3·39	8·8	10·5	2
573	Anon.	20 49 40	+ 19 3	·625	180·0	3·89	9·5	9·9	1
				·625	180·6	3·84	9·4	10·0	1
574	+19° 4628	21 4 49	+ 20 5	·612	47·4	2·93	9·2	9·8	1
				·612	47·7	2·88	9·1	9·6	1
575	Anon.	5 37	+ 20 10	·612	173·0	5·00	9·4	10·3	1
				·612	174·9	5·15	9·4	9·8	1
576	Lpz. II. 10613	8 28	+ 7 25	·590	231·8	2·63	9·0	9·5	1
				·590	230·0	2·46	9·2	9·5	1
577	Berl. A 8638	9 16	+ 17 42	·636	25·2	2·29	9·0	10·0	1
				·636	23·6	2·27	9·0	10·3	1
578	Anon.	12 59	+ 13 17	·628	290·8	3·89	9·4	11·8	1
				·628	293·2	3·82	9·4	11·6	1
579	Anon.	21 25 11	+ 18 45	·636	178·9	3·69	9·4	9·5	1
				·636	177·8	3·84	9·5	9·6	1
580	+11° 4803	22 23 57	+ 12 0	·776	108·3	4·17	9·2	9·6	1
				·776	112·8	4·14	9·1	9·6	1
581	+13° 5082	23 11 54	+ 14 4	·776	20·1	1·38	9·0	9·0	1
				·776	21·7	1·39	9·1	9·1	1
582	+ 3° 4884	23 34 26	+ 3 52	·754	198·2	2·98	9·0	10·3	1
				·754	200·2	2·95	9·0	10·8	1
583	+11° 52	0 19 29	+ 12 1	·776	241·2	3·63	9·0	10·0	1
				·776	244·6	3·52	9·1	10·0	1
584	Berl. A 126	26 32	+ 17 26	·773	172·6	3·22	8·8	11·8	1
585	Berl. A 189	37 39	+ 17 25	·774	210·1	1·16	8·7	9·1	2
				·774	213·0	1·21	8·8	9·2	2
586	+17° 128	0 53 11	+ 18 3	·773	222·7	1·31	9·2	9·2	1
				·773	224·1	1·38	9·3	9·3	1
587	Anon.	1 33 54	+ 22 8	·773	30·6	1·45	9·2	9·4	1
				·773	30·9	1·47	9·2	9·3	1
588	Anon.	2 26 13	+ 19 51	·399	152·1	4·67	9·3	9·4	2
				·754	149·8	4·81	9·3	9·4	1
589	+14° 909	5 21 53	+ 14 52	·754	191·4	5·00	9·3	9·5	1
				·754	190·1	5·11	9·4	9·6	1
590	Anon.	5 25 44	+ 22 18	·754	160·8	4·46	9·4	11·8	1
				·754	160·3	4·64	9·4	12·0	1
591	Anon.	6 11 12	+ 38 53	·644	186·9	5·00	9·5	11·0	1
592	+ 9° 1165	6 11 40	+ 9 11	·773	173·2	3·48	9·0	10·5	1
				·773	170·6	3·60	9·1	10·8	1

J.	Catalogue.	Æ. 1920.	Dec.	Date.	P.	D.	Grandeurs.		n.
		h m s	° ′	1911	°	″			
593	Anon.	6 13 11	+ 37 53	·644	192·2	4·77	9·2	10·2	1
				·644	185·9	4·56	9·3	10·4	1
594	Anon.	20 10	+ 11 9	·773	96·5	2·33	9·5	11·8	1
				·773	93·3	2·60	9·5	11·6	1
	AC			·773	297·3	6·97	9·5	13·0	1
				·773	295·4	6·90	9·5	12·8	1
595	+11° 1173	21 55	+ 11 30	·773	39·1	4·98	9·2	9·2	1
				·773	37·9	5·02	9·3	9·3	1
596	+ 2° 1348	36 53	+ 2 15	·773	43·7	4·93	9·2	9·2	1
597	+ 2° 1351	37 11	+ 2 11	·773	46·5	4·97	9·0	9·0	1
598	Anon.	37 53	+ 2 30	·773	25·8	3·92	9·5	11·9	1
599	Anon.	49 27	+ 14 58	·773	111·5	2·19	9·4	11·7	1
600	+15° 1377	6 51 23	+ 14 57	·773	60·8	3·45	9·0	9·7	1
				·773	60·4	3·40	9·0	9·7	1

Hem:
le 14 *Octobre* 1911.

The Spectrum of Nebulium. By J. W. Nicholson, M.A., D.Sc.

In an account of a possible view of the constitution of matter, communicated to the British Association at Portsmouth, the existence of certain elements with the simplest possible type of atom was taken for granted. One of these was provisionally called nebulium, and identified with the unknown source of some, or perhaps all, of the nebular spectrum lines which cannot be ascribed to hydrogen or helium. The present paper is devoted to a preliminary examination of this hypothesis, and the results which are obtained appear to indicate that a dynamical system which can give rise to these spectral lines has been found.

The main conception involved in the structure of these simple atoms is that of the nature of positive electricity. This is supposed to exist in small spherical volume distributions of uniform density, whose radius is small in comparison even with that of an electron, a reversal of the more generally accepted view. The mass of these positive units is very large in comparison with that of an electron, and gives rise to nearly the whole mass of an atom. The atom is called "simple" when it contains only one positive unit and a single ring of electrons rotating round it.

The atom of nebulium, a hypothetical element predicted by the theory, when electrically neutral, contains four electrons, each with a charge—e—rotating uniformly at equal distances in a circle round a positive nucleus whose charge is $4e$. If one electron be missing, the other three can take up equidistant positions and rotate in a new orbit, the system then consisting of an atom of

nebulium with a single positive charge. In a similar manner, the atom may take up more electrons and acquire a negative charge. But it is still to be regarded as nebulium if the positive charge remains the same. We shall adopt the usual view that the nebulæ are in a state of electrical excitement rather than of high temperature, so that atoms of the gases composing them are continually losing and gaining electrons. It is therefore to be expected that the spectrum of one of these gases should exhibit lines corresponding to the vibrations of a neutral atom, of an atom with one or more positive charges, and of the same atom with negative charges.

From this point of view, we proceed to examine the vibrations of the atom of nebulium. The electrons in steady motion must lie in one plane, in order that their energy may not be dissipated by rapid radiation. Vibrations of the dynamical system which the atom constitutes may take place either in or perpendicular to this plane. We propose in this paper to consider mainly the vibrations perpendicular to the plane, since they appear to be the more important set for the visible spectrum. The investigation of other vibrations presents also some special difficulties, which are more conveniently treated separately.

The greater part of the necessary analysis is an adaptation of that given by Sir J. J. Thomson* in his investigation of the free periods of another type of system, and the formulæ are therefore developed somewhat briefly. Similar analysis by a somewhat different method has been given by Nagaoka,† to whose model of the atom these elementary simple substances closely conform.

Let there be n electrons of charge e in steady circular motion round a positive nucleus of charge νe, the radius of the orbit being a. An agency causes vibrations of the system perpendicular to its plane, and we require the frequencies of the possible types of small oscillation.

Let the system be referred to cylindrical polar coordinates (r, θ, z), where z is perpendicular to the plane, and (r, θ) are polar coordinates in the plane, the origin being at the centre of the undisturbed orbits. An electron is acted upon by an attraction $\nu e^2/(\text{distance})^2$ towards the positive charge and a repulsion $e^2/(\text{distance})^2$ from each of the other electrons. We require the components of these forces perpendicular to the plane.

The force exerted by the s^{th} electron on the p^{th}, resolved perpendicular to the plane, is, if R_{ps} be their distance apart,

$$e^2(z_p - z_s)/R_{ps}^3;$$

and since, for small vibrations, the angle between them is

$$\psi = 2(p - s)\pi/n,$$

we have

$$R_{ps} = 2a \sin \psi,$$

* *Phil. Mag.*, March 1904. † *Ibid.*, May 1904.

so that the force on the p^{th} electron is

$$(e^2/8a^3)(z_p - z_s)\ \text{cosec}^3\,\psi$$

along z increasing. The total force along z, exerted on the p^{th} electron by all the others, is accordingly

$$Z_p = D - \Sigma z_{p+s} D_s \quad . \quad . \quad . \quad . \quad (1)$$

where

$$8a^3 D = e^2 \Sigma^{n-1} \text{cosec}^3 \frac{s\pi}{n}, \qquad 8a^3 D_s = e^2\ \text{cosec}^3 \frac{s\pi}{n} \quad . \quad (2)$$

Let ζ be the displacement of the positive charge and M its mass, m being the mass of an electron. The corresponding force on the electron, due to the positive charge, is

$$-\nu e^2 (z_p - \zeta)/a^3$$

and the attraction of the electrons on the positive nucleus, tending to increase ζ, is

$$\Sigma_1^n \nu e^2 (z_s - \zeta)/a^3.$$

The equations of motion of the nucleus and the electrons therefore become

$$a^3 M\ddot{\zeta} = \nu e^2 \Sigma_1^n (z_s - \zeta) = -\nu n e^2 \zeta + \nu e^2 (z_1 + z_2 + \ . \ . \ . \ + z_n) \quad (3)$$

$$a^3 m \ddot{z}_p = a^3 Z_p - \nu e^2 (z_p - \zeta) \quad . \quad . \quad . \quad (4)$$

If the motion has a period $2\pi/q$, so that all quantities are proportional to e^{iqt}, the first of these equations yields

$$\nu e^2 \zeta = \alpha (z_1 + z_2 + \ . \ . \ . \ + z_n) a^3 \quad . \quad . \quad . \quad (5)$$

where

$$\alpha = \left(\frac{\nu e^2}{a^3}\right)^2 \Big/ \left(\frac{\nu n e^2}{a^3} - Mq^2\right) \quad . \quad . \quad . \quad (6)$$

Now the equation of motion of the p^{th} electron gives

$$-mq^2 z_p = D z_p - \Sigma D_s z_{p+s} - \frac{\nu e^2}{a^3} z_p + \frac{\nu e^2}{a^3} \zeta,$$

and eliminating ζ with (5),

$$z_p \left\{ \frac{\nu e^2}{a^3} - D - mq^2 + \alpha \right\} + \Sigma (\alpha + D_s) z_{p+s} = 0. \quad . \quad . \quad (7)$$

This set of equations, n in number, may be solved by writing

$$z_2 = \omega z_1, \qquad z_3 = \omega z_2, \quad . \quad . \quad . \quad . \quad (8)$$

where ω is an n^{th} root of unity, provided that

$$\frac{ve^2}{a^3} - D - mq^2 + (\omega D_1 + \omega^2 D_2 + \ldots) + a(1 + \omega \ldots + \omega^{n-1}) = 0 \quad (9)$$

But the coefficient of a is zero, so that the mass of the positive nucleus disappears from the period equation. Again,

$$\omega D_1 + \omega^2 D_2 + \ldots = (e^2/8a^3)\sum_{s=1}^{n-1} \cos 2\kappa s \frac{\pi}{n} \operatorname{cosec}^3 \frac{s\pi}{n},$$

where ω has been written as $\cos 2\kappa\frac{\pi}{n} + \iota \sin 2\kappa\frac{\pi}{n}$, κ having all integral values from unity to $n-1$. Writing

$$P_\kappa = \sum_1^{n-1} \cos 2\kappa \frac{s\pi}{n} \operatorname{cosec}^3 \frac{s\pi}{n} . \quad . \quad . \quad (10)$$

the period equation may be expressed in the form

$$mq^2 = e^2(8\nu + P_\kappa - P_0)/8a^3 \quad . \quad . \quad . \quad (11)$$

noticing that $D = P_0$. All the periods are obtained by giving κ the sequence of values 0, 1, 2, . . . $n-1$. We shall call κ the "class" of the vibration. It may be positive or negative, a change of sign of κ not altering P_κ, and therefore q^2.

The work of Schott* on the intensities of lines produced by a single ring of electrons indicates that vibrations of class ± 3 or higher will not ordinarily appear in the visible spectrum. But it is noteworthy that the mechanism of production of spectra contemplated in this paper is not quite identical with that of Schott in his consideration of ionised rings. His conclusions in this respect should nevertheless apply to the present case.

The nature of the problem.

We have considered only the forces of electrostatic origin. In reality the problem is one of electro-magnetics, and the magnetic effects should strictly be included. But if the rotation of the ring is not too rapid in comparison with the velocity of light, the difference is small. For small rotations, the magnetic force due to the ring will be mainly axial, and will produce no mechanical effect on an electron vibrating in that direction. We may therefore expect the period equation deduced above to be very accurate if an investigation of the tangential velocity of an electron in its orbit gives a sufficiently low value.

It will be found that the error in our calculated wave-length is generally only about 3 or 4 tenth-metres, and in a later section it will be shown that this is exactly the magnitude of the error to be expected. A concrete idea of the size of the atom will at the same time be given.

* *Phil. Mag.*, Feb. 1907, and other papers.

The functions P_κ.

In subsequent work we shall require the values of P_κ for various values of κ, the class of the vibration, and of n the number of electrons. These have been used by Sir J. J. Thomson as far as $n=6$. The values are:—

$n=2$

$P_0=1,\ P_1=-1$

$n=3$

$$P_0=16/3\sqrt{3},\ P_1=-8/3\sqrt{3}=P_2 \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$n=4$

$P_0=1+4\sqrt{2},\ P_1=-1,\ P_2=1-4\sqrt{2}$

$n=5$

$P_0=12{\cdot}1732,\ P_1=1{\cdot}1609,\ P_2=-7{\cdot}249$

$n=6$

$P_0=17+4/3\sqrt{3}, P_1=7-8/3\sqrt{3}, P_2=-7-8/3\sqrt{3}, P_3-17+16/3\sqrt{3}$

The notation adopted is somewhat different. Thomson's P_κ includes the factor $e^2/8a^3$.

Periods of the neutral atom of nebulium.

For the neutral atom, with no resultant charge, $n=4$, $\nu=4$, and the period equations are

$$ma^3q^2=e^2\left\{\nu+(P_\kappa-P_0)/8\right\}\ \kappa=0,\ 1,\ 2,\ 3.$$

Values of κ higher than 2 only cause repetition of periods. Thus with the values of the P functions already quoted,

$$ma^3q^2=e^2\left\{4,\ 4-(2+4\sqrt{2})/8,\ 4-\sqrt{2}\right\} \quad . \quad (13)$$

the second being repeated.

Let ω be the angular velocity of the ring of electrons in steady motion. This will cause no confusion with the ω, an n^{th} root of unity, employed earlier. The radius of the orbit is a, and therefore $ma\omega^2$ must be the normal force towards the centre, acting on an electron. The positive nucleus gives a radial attraction $\nu e^2/a^2$, and the combined action of the other electrons gives a radial repulsion $e^2S_n/4a^2$, where

$$S_n=\Sigma_1^{n-1}\ \text{cosec}\ \frac{s\pi}{n} \quad . \quad . \quad . \quad . \quad (14)$$

Accordingly,

$$ma\omega^2=\frac{e^2}{a^2}(\nu-S_n/4)$$

or

$$e^2/ma^3=\omega^2/(\nu-S_n/4) \quad . \quad . \quad . \quad (15)$$

is the equation for the steady state of a charged or uncharged simple atom. When $n=4$, we find $S_n = 1+2\sqrt{2}$. Thus for the neutral atom of nebulium the period equations become

$$q^2/\omega^2 = 16/(15-2\sqrt{2})$$

$$q^2/\omega^2 = 1$$

$$q^2/\omega^2 = 8/(8+\sqrt{2})$$

corresponding to $\kappa=0$, $\kappa=1$ or 3, $\kappa=2$ respectively. Finally,

$$q/\omega = \left\{ 1{\cdot}146533, \quad 1, \quad {\cdot}849778 \right\} \quad . \quad . \quad (16)$$

Periods relatively to a stationary observer.

It is important to remember that the periods calculated are those of waves relative to the motions of the electrons in the ring. A wave of class κ travels round the ring with velocity q/κ, and the frequency manifested to an observer is not q, but $q+\kappa\omega$, where κ may be positive or negative according to the sense in which the wave travels round, whether with the general steady motion of the electrons or against it. This consideration has usually been overlooked, but was emphasised by Schott in his discussion of Nagaoka's theory of the band and line spectra produced by an atom of this nature with a large positive nucleus. For positive values of κ, the frequencies with reference to the observer become in the present case

$$q'/\omega = \left\{ \underset{\kappa=0}{1{\cdot}146533}, \quad \underset{\kappa=1}{2}, \quad \underset{\kappa=2}{2{\cdot}849778}, \quad \underset{\kappa=3}{4} \right\}$$

and for negative values there is a set of frequencies

$$q''/\omega = \left\{ \underset{\kappa=0}{1{\cdot}146533}, \quad \underset{\kappa=-1}{0}, \quad \underset{\kappa=-2}{1{\cdot}150222}, \quad \underset{\kappa=-3}{2} \right\}$$

where negative values have been written as positive, for the sign is not important in a consideration of frequencies. The whole set, excluding $\kappa=0$, becomes

$$q/\omega = \left\{ 1{\cdot}146533, \quad 1{\cdot}150222, \quad 2, \quad 2{\cdot}849778, \quad 4 \right\}$$

with corresponding wave-lengths

$$\lambda = 2\pi C/\omega \left\{ {\cdot}872194, \quad {\cdot}86939, \quad {\cdot}5, \quad {\cdot}3509, \quad {\cdot}25 \right\} \quad (17)$$

where C is the velocity of light.

We may expect that the first of these vibrations should be strong, and the second perhaps too weak to be observed. If the

first or second is in the visible spectrum, the other three cannot be, so that they need no further consideration, and attention may be limited to the first pair, of classes zero and -2, the second being perhaps invisible. In order to identify the first, we consider, in addition, the principal vibration, of class zero, of the atom in its own plane. For any number of electrons and any strength of nucleus, this has a frequency ω if the velocity of the electrons is not comparable with that of light. The proof of this result is very simple, but may be deferred at present, to take its place in the general investigation, of a later paper, of the periods of the atom for these vibrations.

This vibration is of class zero, and therefore ω is also its period as manifested to the observer. Its wave length is $2\pi C/\omega$. We therefore have three vibrations capable of appearance in the visible spectrum, of wave-lengths

$$\lambda = 2\pi C/\omega \left\{ 1, \quad \cdot 872194, \quad \cdot 86939 \right\} \quad . \quad (18)$$

and the first two should be strong.

We must now show that the ratio of these two is identical with that of two important nebular lines. Consider the chief nebular line $\lambda = 5006\cdot9$ and the line $\lambda = 4363\cdot4$. These are the values finally adopted by W. H. Wright,* whose system of nebular lines has been used throughout this paper for purposes of comparison. The second line has always an intensity about 1/10 of that of the first, a fact which may indicate a common origin.

Assuming that the first of our lines is the line $\lambda = 5006\cdot9$, the second should have a wave-length $5006\cdot9 \times \cdot872194 = 4367\cdot0$, with an error of only $3\cdot6$ tenth-metres.

The proportional error is $3\cdot6$ in about 4400, or $\cdot0008$ in unity.

Degree of approximation, and the size of the atom.

The neglect of the magnetic effects of the motion, which has already been pointed out in certain phases, can all be comprised under one heading. The error consists of the neglect of the velocity of an electron in comparison with that of light. It is therefore comparable in magnitude at most with $a\omega/C$. Now in the above identification of the lines we have written

$$2\pi C/\omega = 5007 \text{ tenth-metres} = 5.10^{-5} \text{ cms.}$$

so that

$$\omega/C = 1\cdot255.10^{5} \quad . \quad . \quad . \quad . \quad (19)$$

giving the angular velocity in the atom. Moreover, for the steady motion

$$\omega^2 = e^2(\nu - S_n/4)/ma^3 = 3\cdot0429e^2/ma^3 \quad . \quad . \quad (20)$$

* *Lick Observatory Bulletin*, No. 19; *Astrophys. Journal*, xvi. (1902).

where m is the mass of a slowly moving electron, given by $e/m\mathrm{C} = 1{\cdot}88.10^{7}$, and $e = 3{\cdot}4.10^{-10}$. The former value is due to Kaufmann, and the latter to J. J. Thomson. Accordingly, on reduction,

$$\omega^2 a^3 = 5{\cdot}835.10^{8},$$

and with the value already found for ω,

$$a^3 = 41{\cdot}18\ 10^{-24}$$

or

$$a = 3{\cdot}453\ 10^{-8} \quad . \quad . \quad . \quad . \quad (21)$$

This is the radius of the atom of nebulium, and it is in complete accord with current estimates of atomic radius, which give 10^{-8} as representing the order of magnitude.

Since $\omega = 1{\cdot}255.10^{5}$ C, the velocity of an electron is

$$a\omega = 4{\cdot}33.10^{-3}\mathrm{C} \quad . \quad . \quad . \quad . \quad (22)$$

and (velocity of electron)/(velocity of light) = ·00433, which is very small. The error in the present investigation cannot therefore exceed about 4 parts in a thousand, even if first order terms of $a\omega/\mathrm{C}$ actually occur, so that it cannot be more than a few tenth-metres. This is a sufficient margin of accuracy, and the present investigation is sufficient, without the higher approximation, to show that nebulium can be the source of most of the nebular lines. The error will actually be rarely as large even as 4 tenth-metres.

The system of nebular lines.

Before proceeding further, it is convenient to reproduce Wright's final table of wave-lengths of the nebular lines. When a line is ascribed to hydrogen or helium, the fact is stated in the neighbouring column. Some of these identifications have been disputed frequently, especially in the case of certain hydrogen lines.

The table, which is the result of the examination of the Orion nebula, and of the nebulæ G.C. 4390, G.C. 4964, N.G.C. 7027, contains eleven lines of unknown origin, and there is some reason for the belief that they may all be due to the same substance. Considerations of relative intensity are frequently of doubtful value, quite apart from the well-known effect of purely instrumental conditions on the intensities which are actually observed. Moreover, it must be remembered, as Schott, for example, has pointed out, that an apparently constant ratio of brightness of two lines does not necessarily mean that they are produced simultaneously by the same type of atom. It may mean, on the contrary, that they are produced under the same conditions. For example, a vibration of a given class may be specially excited under certain conditions; and if it is produced

both in a neutral atom and an ionised one, it is possible for the pair of vibrations to have a constant intensity ratio from one nebula to another, whereas the remaining vibrations of the neutral atom might have different ratios to that of the given class in nebulæ whose conditions were different.

Constant ratio of brightness of two lines is therefore in general an insufficient criterion for their common origin, although, in the absence of other knowledge, it gives a useful mode of attack, always possessing a certain balance of probability in its favour. It has been used already in the paper for a case with which it is in accord, but only as secondary evidence. As primary evidence its value would be subject to doubt.

The investigation of the last section, of the order of magnitude of the error, has confirmed the conclusion developed in that preceding, namely, that the principal nebular line 5006·9 and the line 4363·4 are in accordance with the theory that they are due to the neutral atom of nebulium. The ratio of wave-lengths is almost exactly correct. We shall find a similar correspondence for other pairs, and even trios, of lines. Moreover, these lines in the spectrum of the neutral atom ought theoretically to be the strongest set, and they are both strong in the nebulæ, the former being the strongest of all the lines.

The whole system of lines given by W. H. Wright is:—

Nebulæ.	Vacuum Tubes.	Nebulæ.	Vacuum Tubes.
3726·4	...	4101·91	4101·89 H_δ
3729·0	...	4340·62	4340·63 H_γ
3835·8	3835·6 H_η	4363·37	...
3868·88	...	4471·71	4471·65 helium
3889·14	3889·15 H_ζ	4685·73	...
3965·1	3964·9 helium	4740·0	...
3967·65	...	4861·54	4861·50 H_β
3970·23	3970·25 H_ϵ	4959·05	...
4026·7	...	5006·89	...
4068·8	...		

Now the case $\kappa = -2$ for the neutral atom has been seen to lead to another line which will probably be very weak. Its wave-length should be $5006{\cdot}9 \times {\cdot}86939 = 4352{\cdot}9$. It does not appear in Wright's table, and the important vibrations of the neutral atom are therefore those of class zero, as may be expected. In cases of ionisation, vibrations of higher classes can sometimes be important, as Schott has shown; and this fact will be in accordance with later results of the paper.

Nebulium with a single negative charge.

Consider now the same atom to which an extra electron has become attached. The five electrons will take up equidistant

positions and rotate differently. The nucleus retains its charge $4e$, so that $\nu = 4$, $n = 5$. The angular velocity is given by

$$ma^3\omega^2 = e^2(\nu - S_5/4) = 2{\cdot}6236e^2 \quad . \quad . \quad (23)$$

since the value of S_5 is found to be 5·5056.

The period equations are

$$ma^3q^2 = e^2\left\{\nu + (P\kappa - P_0)/8\right\} \kappa = 0, 1, 2, \ldots .$$

But when

$$n = 5,\ P_1 - P_0 = -11{\cdot}0123,\ P_2 - P_0 = -19{\cdot}4222,\ P_3 = P_2,\ P_4 = P_1$$

so that

$$q^2 = e^2\left\{4, \quad 2{\cdot}62346, \quad 1{\cdot}5722\right\}/ma^3$$

$$= \omega^2\left\{1{\cdot}5246, \quad 1, \quad {\cdot}59925\right\}$$

or

$$q/\omega = \left\{\begin{matrix} 1{\cdot}2347, & 1, & {\cdot}77411 \\ \kappa = 0 & \kappa = 1,4 & \kappa = 2,3 \end{matrix}\right\} \quad . \quad (24)$$

These are the relative frequencies. The frequencies estimated by an observer, for positive values of κ, are given by

$$q'/\omega = \left\{\begin{matrix} 1{\cdot}2347, & 2, & 2{\cdot}77411, & 3{\cdot}77411, & 5 \\ \kappa = 0 & \kappa = 1 & \kappa = 2 & \kappa = 3 & \kappa = 4 \end{matrix}\right\}$$

and for negative values by

$$q''/\omega = \left\{\begin{matrix} 1{\cdot}2347, & 0, & 1{\cdot}22589, & 2{\cdot}22589, & 3 \\ \kappa = 0 & \kappa = -1 & \kappa = -2 & \kappa = -3 & \kappa = -4 \end{matrix}\right\}$$

There is also a frequency $q = \omega$ for vibrations in plane. Selecting only those in the visible spectrum with $q = \omega$, we have the set

$$q/\omega = \left\{1{\cdot}2347, \quad 1, \quad 1{\cdot}22589\right\}$$

with corresponding wave-lengths

$$\lambda = 2\pi C\left\{{\cdot}80991, \quad 1, \quad {\cdot}81573\right\}/\omega \quad . \quad . \quad (25)$$

and the last, corresponding to $\kappa = -2$, can in this case be important. We find that three lines have been observed in accordance with these wave-lengths. If the line $\lambda = 3869$ be identified with the last of the three, the first should have a wave-length equal to 3869/·81573 or 4743. This differs only by 3 tenth-metres from the nebular line $\lambda = 4740$, the error being practically the same as before.

Again, the second vibration should have a wave-length equal to $4740 \times \cdot 80991$ or 3838·97. This differs from the line 3835·8, usually identified as H_η, by 3·2 tenth-metres, an error nearly the same.

There has been some discussion as to whether this line is really H_η or a composite line. For example, in his paper on a possible series in the nebular spectrum, Love * gave reasons for concluding that it was not a hydrogen line. Wright † maintained that it had only been observed in the Orion nebula where the hydrogen series was comparatively bright. Without expressing a definite opinion on the point, we may notice the suspicious circumstance that the final value deduced by Wright for this line is 3835·8, differing by ·2 from the hydrogen line H_η in a vacuum tube. In no other case is there a difference greater than ·04 between corresponding hydrogen lines in the nebula and the vacuum tube in Wright's investigation. For this line, if hydrogen, the difference is five times as large, and it does seem possible that the line is not due to hydrogen. We shall provisionally accept it as a composite line of hydrogen and nebulium.

The errors have never been greater, up to the present, than 3·5 tenth-metres, and are approximately constant, thus indicating a common cause, in the neglect of v/C, where v is the velocity of an electron.

Nebulium with a single positive charge.

We pass now to a consideration of an atom which has lost an electron, the remaining three taking up new equidistant positions. For this case, $\nu = 4$, $n = 3$, and the period equations are as usual

$$q^2 = e^2 \left\{ 8\nu + (P_K - P_0) \right\} / 8ma^3$$

$$= e^2 \left\{ 4, \quad 4 - 1/\sqrt{3}, \quad 4 - 1/\sqrt{3} \right\} / ma^3 \quad . \quad (26)$$

Since $S_3 = 4/\sqrt{3}$, the angular velocity is given by

$$\omega^2 = e^2 (4 - 1/\sqrt{3}) / ma^3 \quad . \quad . \quad . \quad (27)$$

and therefore

$$q^2/\omega^2 \left\{ \underset{K=0}{1{\cdot}16873}, \quad \underset{K=1}{1}, \quad \underset{K=2}{1} \right\}$$

giving the relative vibrations

$$q/\omega = \left\{ 1{\cdot}0811, \quad 1, \quad 1 \right\}.$$

The complete set of vibrations relative to the observer is

$$q/\omega = \left\{ 1, \underset{K=0}{1{\cdot}0811}, \underset{K=1}{2}, \underset{K=2}{3}, \underset{K=-1}{0}, \underset{K=-2}{1} \right\}$$

* *Monthly Notices*, vol. lxii. p. 526. † *Ibid.*, vol. lxii. p. 632.

and the wave-lengths to be expected in the visible spectrum are

$$\lambda = 2\pi C \left\{ 1, \cdot 9250 \right\} / \omega \quad . \quad . \quad . \quad (28)$$

Let us identify the first line with the supposed helium line found by Wright at $\lambda = 4026 \cdot 8$. This is a somewhat doubtful helium line, and it is significant that the discrepancy between Wright's table and the vacuum tube measures is, for this line, as much as ·4, twice as great even as that in the case of H_η already discussed. It may be strongly urged that this line is not due to helium.

Making the identification suggested, the wave-length of the second line should be $4026 \cdot 8 \times \cdot 9250 = 3724 \cdot 8$.

Wright gives two nebular lines at $\lambda = 3726 \cdot 4$ and $\lambda = 3729 \cdot 0$, exhibiting differences from the present value of only 1·6 and 4·2 respectively. It seems certain that the line is one component of this doublet, but it is not possible to choose between the components without the more complete investigation. Probably the second is the proper component, as it gives the more usual error, though a larger one.

Of the eleven nebular lines, we have now accounted for five, with the addition of the supposed H_η and helium lines.

Nebulium with a double positive charge.

Consider now the nebulium nucleus with only two electrons, so that

$$\nu = 4, n = 2, P_0 = 1, P_1 = -1$$

The equation of steady motion is

$$\omega^2 = 15e^2/4ma^3 \quad . \quad . \quad . \quad . \quad (29)$$

and

$$q^2 = e^2(4, 3 \cdot 75)/ma^3 \quad . \quad . \quad . \quad (30)$$

or

$$q/\omega = \left\{ 1 \cdot 03277, 1 \right\}$$

with the frequency $q = \omega$ added, the only frequencies in the visible spectrum manifested to an observer become

$$q/\omega = \left\{ \begin{matrix} 1, 1 \cdot 03277 \\ \kappa = 0 \end{matrix} \right\} \quad . \quad . \quad . \quad (31)$$

of wave-lengths

$$\lambda = 2\pi C \left\{ 1, \cdot 96826 \right\} / \omega \quad . \quad . \quad . \quad (32)$$

Let us suppose that the first line is identical with the nebular

line $\lambda = 3967{\cdot}6$, to which special interest attaches, as it was believed to be the line H_ϵ until Wright isolated it as a separate vibration not due to hydrogen. Then the second line should have a wave-length 3841·5, differing from H_η, already classed as complex, by only about 5 tenth-metres. It would seem, therefore, that H_η, whether it contains a hydrogen line or not, should be a faint double line of nebulium when it occurs in nebulæ.

Apparently, therefore, the line 3967·6 isolated by Wright is due to nebulium with a double positive charge.

Nebulium with two negative charges.

Suppose that atoms are present with six electrons, so that $\nu = 4$, $n = 6$,

$$S_n = 5 + 4/\sqrt{3},\ P_1 - P_0 = -10 - 8/\sqrt{3},\ P_2 - P_0 = -24 - 8/\sqrt{3},$$
$$P_3 - P_0 = -34,\ P_4 = P_2,\ P_5 = P_1.$$

In steady motion,

$$ma^3\omega^2 = e^2(2{\cdot}1725). \quad . \quad . \quad . \quad (33)$$

and the period equations become

$$ma^3q^2 = e^2 \left\{ \begin{array}{cccc} 4, & 2{\cdot}1725, & {\cdot}4225, & \text{negative} \\ \kappa = 0 & \kappa = 1, 5 & \kappa = 2, 4 & \kappa = 3 \end{array} \right\} . \quad (34)$$

Thus one of the periods is imaginary, and the mode is unstable. But it is the mode $\kappa = \pm 3$, which, as Schott has shown, always has a very small amplitude for a given type of disturbing agency, and the atom with 6 electrons might therefore persist for a considerable time and be constantly formed again, if broken up, by the capture of stray electrons. It is quite possible, therefore, that the lower modes of this system should give rise to spectrum lines in a mass of gas under high electrical excitement.

We find ultimately, for the relative frequencies,

$$q/\omega = \left\{ \begin{array}{cccc} 1{\cdot}3569, & 1, & {\cdot}44100, & \text{imaginary} \\ \kappa = 0 & \kappa = 1, 5 & \kappa = 2, 4 & \kappa = 3 \end{array} \right\} . \quad (35)$$

giving possible frequencies relatively to an observer, including $q = \omega$, of

$$q/\omega = \left\{ \begin{array}{ccccccc} 1, & 1{\cdot}3569, & 1{\cdot}55900, & 2, & 2{\cdot}44100, & 3 \\ & \kappa = 0 & \kappa = -2 & \kappa = 1 & \kappa = 2 & \kappa = 3, -3 \end{array} \right\}$$

where the value 3 is obtained as the real part of the complex period, which would thus manifest a frequency if the unstable vibration were strong enough. It is to be noted that this instability may not be absolute, but may be due to the supposition that the displacement is small.

The corresponding wave-lengths are

$$= 2\pi C \left\{ 1, \quad \cdot 73698, \quad \cdot 64144, \quad \cdot 5, \quad \cdot 40967, \quad \cdot 3333 \right\} / \omega \quad . \quad (36)$$

There are two lines in the nebular spectrum whose ratio is approximately correct. Identify the fourth line with the "second nebular line" $\lambda = 4959$. Then the fifth should have a wave-length $4959 \times \cdot 81934$ or $4063 \cdot 2$. This differs from the line $4068 \cdot 8$ by only $5 \cdot 6$ tenth-metres. At the same time the last line is precluded from appearance in the visible spectrum of Wright's range, as are also the others.

It is not surprising that the line $\kappa = 1$ should be so intense, for it is a case of ionisation, and, moreover, the unstable mode, even if it does not cause the atom to break up and lose an electron, may cause greater radiation of energy in the more stable modes.

The remaining lines.

We have now accounted for all the lines given by Wright except two, namely, the line $\lambda = 4685 \cdot 7$, and the other component of the doublet near 3728. It seems possible that these are also due to the same substance, and they may correspond to stable vibrations of the atom in its own plane, although the majority of these, so far as an investigation now in progress has yet indicated, seem to lie outside the visible spectrum. The second line is notably near a line calculated by Rydberg as belonging to Hydrogen.

In every case but one, that of H_η, it has never been necessary to use any line twice. This appears to preclude accidental coincidences, which would be sufficiently remarkable in any case, for no necessary ratio of wave-lengths has failed to be represented among the very small number of nebular lines, with a small and very nearly constant error of the proper amount.

A more exact determination of the theoretical wave-lengths is laborious, and outside the scope of the present paper. But it may be claimed, from the preliminary results in this paper, that the hypothetical atom of nebulium described at the beginning of the paper is capable of accounting for at any rate the greater part of the nebular spectrum. It may be shown that atoms of a similar kind can explain the spectrum of the solar corona equally well.

Summary.

The results of the investigation are shown in the following table. Nu is the symbol for nebulium, the signs + and − denoting the electrical charges which they contain, Nu itself denoting the neutral atom.

Nebular Line.	Identification.	Nebular Line.	Identification.
3726·4	Nu+	4101·91	H_δ
3729·0	...	4340·62	H_γ
3835·8	H_η, Nu−, Nu++, (?)	4363·37	Nu
3868·88	Nu−	4471·71	Helium
3889·14	H_ζ	4685·73	...
3965·1	Helium	4740·0	Nu−−
3967·65	Nu++	4861·54	H_β
3970·23	H_ϵ	4959·05	Nu−−
4026·7	Helium ?, Nu+	5006·89	Nu
4068·8	Nu−−	...	...

General Considerations.

It will be noticed that a connection has not been formulated between the various sets of wave-length ratios. There are two reasons which render this impossible at present without a very long discussion. In the first place, some mode by which the atom may be made of definite structure, in order to account for the fineness of spectral lines, must be invoked. This question has been discussed by Rayleigh and Jeans; and subsequently Schott, in the papers already cited, has suggested a method by which this result may be achieved, with the help of expanding electrons, in the case of an atom made up of rings of electrons rotating under the influence of a controlling agency, such, for example, as the positive nucleus of the present paper. This conception of expanding electrons is of great service in the possible elucidation of many phenomena, perhaps even of gravitation; but it is sufficient for the present purpose to note that the atomic systems which we contemplate may give fine spectral lines.

But in order to relate different systems to one another, it is necessary to obtain a measure of the energy of such systems, and of the energy imparted to a system by, for example, the arrival of another electron. For this purpose, a sufficiently accurate value of e^2/m, to about four significant figures, is essential, in order to obtain the relations between the spectral lines of different systems. This value cannot be obtained experimentally with the necessary accuracy, and its indirect determination on theoretical grounds from the available data in other branches of physics is difficult.

In these circumstances, it is perhaps desirable to appeal to the theory of probability for a further confirmation of the results suggested in the paper, and the following investigation, on the simplest possible lines, is therefore appended.

Let us calculate the chance that the coincidences of the ratios of theoretical wave-lengths with those of the nebular lines is fortuitous. The range of spectrum considered is from λ 5007 to λ 3729, that is to say, it extends over 1728 tenth-metres. The total number of lines at our disposal in this region, including the

two which had previously been identified in another manner, is twelve. The usual error in the estimates of wave-lengths by the present method of ratios is about 3 tenth-metres, positive or negative.

Let any ratio be specified at random, limited only by the fact that it must be less than that of the extreme wave-lengths in the range, or, conversely, greater than the ratio reciprocal to this. Evidently an unfavourable case is comprised in this definition, and the probability of fortuitous concurrence will actually be much less than the upper limit calculated on this basis.

Take any line in the range and multiply by this random ratio. The chance that the product should range within 6 tenth-metres about one of the remaining lines, eleven in number, is $11 \times 6/1728$, and as the first line can be selected in twelve ways, the total chance is $6 \times 11 \times 12/1728$. It is, of course, actually less, for the selection of many particular initial lines would carry the product out of the proper range.

We are now left with ten lines, for which three were found to have the proper ratios. Taking another random ratio, the chance that it should be presented is less than $6 \times 10 \times 9/1728$. The chance that the third line of the trio should be within ± 3 tenth-metres of a nebular line is then $6 \times 8/1728$, and therefore the total chance for the trio is $36 \times 8 \times 9 \times 10/(1728)^2$, as an upper limit.

Seven lines remain, and the chance for a pair is $6 \times 7 \times 6/1728$. The chance for another pair in the remaining five is $6 \times 5 \times 4/1728$, and that for a pair in the last three, with a duplication of the line H_η, which was found necessary in this case, is $6 \times 3 \times 11/1728$.

Finally, the chance that the correspondences which have been found are all fortuitous must be considerably less than the product of these chances. This product is 5.10^{-6}, so that the required chance is much less than 5 in a million.

This consideration in itself is a strong confirmation of the validity of the views expressed in the paper, even in the absence of the definite relations among the sets of ratios. When we add the further considerations that, in the first place, an elementary atom of a similar kind, with five electrons when neutral, exhibits complete correspondences of the same kinds, with an identical margin of accuracy, in the solar coronal spectrum, and secondly, that the same atom, and another with three electrons when neutral, cannot give even badly approximate correspondences in the nebular spectrum in more than one or two cases, the demonstration is much more complete, and it is possible to say that the element nebulium as defined is a probable source of the nebular lines.

Trinity College, Cambridge:
1911 *October* 25.

Fifth Note on the Number of Faint Stars with Large Proper Motions. Zone +25°. By F. A. Bellamy, M.A.

1. The former notes on this subject will be found in the *M.N.*, vols. lxix. pp. 57 and 491, lxxi. pp. 45 and 582. The present note is the fifth of the series, and it relates to the comparison of plates in +25° zone (+23° 55′ to +26° 5′). The comparison of all satisfactory pairs of plates that had been actually measured at the date of this paper, and had a difference between the epochs of ten years or more, has now been completed, the necessary money for the preliminary calculations having been provided by the Government Grant Fund.

Other plates exposed more than ten years ago in this and in other zones may ultimately be of use, but at present they have not been measured.

2. All instances where the differences either in x or y amounted to ·005 or 1″·5 (and in a number of cases with ·004 and ·004, or some with ·004 and ·003) have been specially examined and re-measured. The number of stars re-measured on one or both plates has considerably exceeded 2000. A large number of these stars were examined by Mr. W. H. J. Waterfield, B.A. of Christ Church, some others by Professor Turner and Mr. H. C. Plummer; the rest, including a general examination and the collection of the whole material used, has been revised by the writer.

3. It may be remarked that the material available in zone +25° and summarised in Table I. is slightly less than the total amount used in the comparison of the plates used in zones +26°, +27°, +28°, and +29°. The number of stars compared in these four zones was 16,617 on 93 plates, and they yielded 315 stars with total displacements greater than the limits adopted: in zone +25°, 13,577 stars have been compared on 82 plates and 423 stars have shown sensible displacement. This greater number was expected as the mean epoch is 14·41 years as compared with 12·5 years for the previous zones; in +25° many pairs of plates have a difference in epoch of fifteen, sixteen, and some over seventeen years.

Table I. gives a general view of the results. The distribution in R.A. is much more uniform than in either of the other four zones compared; there is at least one plate in every hour of R.A., and there are 15 plates well distributed within the Milky Way.

4. In addition to these 82 plates, 8 other plates with differences between the epochs of only 6·1 and 9·0 years were included in the comparison by an oversight. These 8 plates contained 2534 stars common to the pairs, and yielded 10, 2, and 8 P.Ms. for the three groups; they have been placed in a supplementary list following Table I. They have not been included in the totals discussed in Tables II., III. As most of these 10 stars produced fairly large P.Ms., it was thought advisable to include them in Table IV.

Besides the 423 P.Ms. given in Tables IV. and V., 10 other P.Ms. were independently found on the edges of the plates overlapping in R.A.; these have only been entered once in Table IV., and they are not included in any summaries in this paper.

TABLE I.

Zone +25°. Stars with probable Large P.Ms.

R.A. of Plate Centre.	Approx. Galactic Latitude.	Total Stars compared.	Interval in Years.	Total P.Ms.	Centennial P.Ms. 15″ to 20″.	Centennial P.Ms. >20″.
h m						
0 52	37°	107	15·95	3	2	0
1 16	36	106	15·87	6	1	2
1 24	36	77	15·03	1	1	0
1 32	36	83	16·16	3	0	2
1 48	35	113	16·16	6	0	0
2 4	34	243	16·84	14	1	1
2 28	31	166	16·80	3	0	1
3 8	27	137	17·11	2	0	0
3 16	26	221	17·01	9	0	2
3 24	25	271	17·04	9	1	0
3 32	24	259	17·04	3	0	1
4 52	9	72	14·02	7	2	3
5 8	7	132	15·85	4	1	1
5 16	6	80	11·82	3	1	1
5 24	4	207	9·99	4	3	1
5 32	3	533	10·05	8	7	1
5 40	1	585	15·78	10	2	1
5 56	2	752	10·05	14	11	3
6 4	4°	504	9·97	6	5	1
6 12	6	382	14·97	8	0	2
6 20	8	175	10·17	4	2	2
8 4	28	346	15·96	10	6	2
9 16	45	69	11·92	3	1	0
9 32	47	71	11·90	2	1	1
9 40	49	129	9·92	4	3	1
9 56	52	76	11·95	2	1	1
10 4	54	106	16·88	6	0	1
10 12	56	129	10·86	7	2	3
10 28	60	74	16·88	5	1	0
11 0	66	73	15·88	9	4	1
11 24	71	101	9·81	2	2	0
11 32	73	63	9·95	0	0	0

TABLE I.—*continued.*

R.A. of Plate Centre.	Approx. Galactic Latitude.	Total Stars compared.	Interval in Years.	Total P.Ms.	Centennial P.Ms. 15″ to 20″.	Centennial P.Ms. > 20″.
h m						
11 40	75°	76	9·81	1	1	0
11 48	77	78	9·95	0	0	0
11 56	79	63	16·00	2	0	0
12 4	80	61	15·94	5	0	0
12 12	82	75	10·63	1	1	0
12 20	84	93	16·63	6	0	1
12 28	86	70	16·63	6	0	1
12 36	86	65	15·94	3	0	0
12 44	85	76	16·89	13	0	3
13 0	83	82	16·63	7	0	2
13 8	82	70	16·90	5	1	0
13 16	81	72	16·90	8	1	1
13 24	79	49	16·89	6	0	2
13 32	78	72	16·89	11	4	0
13 40	76	95	16·90	4	1	0
13 48	74	84	16·90	6	3	1
13 56	72	75	16·90	8	0	2
14 4	70	61	15·79	3	1	1
14 20	68	73	10·84	7	3	3
14 28	66	75	15·80	3	0	1
14 36	64	68	15·79	5	1	1
14 44	62	68	15·79	4	3	0
15 0	59	80	9·74	4	3	1
15 8	57	79	15·70	4	0	2
15 16	55	91	15·71	6	1	1
15 32	52	82	15·71	4	0	0
15 40	50	109	9·74	2	1	1
16 4	45	125	15·70	5	0	1
16 28	40	91	16·16	4	0	2
16 52	34	197	16·90	9	0	3
17 0	32	108	16·92	11	1	0
17 8	30	164	10·94	1	0	1
17 16	28	184	16·85	9	3	0
17 48	24	296	16·73	2	0	0
18 12	18	251	16·76	2	0	0
18 20	15	301	15·15	5	2	1
18 36	12	195	16·14	3	0	0
18 52	8	159	12·41	4	3	1

TABLE I.—*continued.*

R.A. of Plate Centre.	Approx. Galactic Latitude.	Total Stars compared.	Interval in Years.	Total P.Ms.	Centennial P.Ms. 15″ to 20″.	Centennial P.Ms. >20″.
h m						
19 8	5°	251	12·41	5	2	1
19 16	4	207	12·11	4	1	1
19 24	3	310	12·13	5	3	1
19 32	2	337	11·13	4	2	1
20 20	6	353	10·14	4	3	1
21 0	15	420	11·03	6	3	0
21 40	21	215	10·95	6	5	1
21 48	23	206	10·97	1	1	0
22 4	24	256	12·03	7	1	3
22 28	28	152	11·93	11	4	3
23 8	33	151	10·87	3	0	0
23 24	34	165	10·87	1	0	1
25° 82 plates		13,578	14·41 (wt. mean)	423	114	81
26° to 29° 93		16,617	12·47 ,,	324	106	80
Totals 175		30,195	13·59 (wt. mean)	747	220	161

Supplementary.

R.A. of Plate Centre.	Approx. Galactic Latitude.	Total Stars compared.	Interval in Years.	Total P.Ms.	Centennial P.Ms. 15″ to 20″.	Centennial P.Ms. >20″.
h m						
6 28	9°	225	6·07	0	0	0
6 36	11	355	6·10	2	0	2
7 0	15	290	6·10	3	0	3
8 20	31	117	6·06	0	0	0
8 52	39	137	6·04	0	0	0
10 52	65	122	8·93	2	2	0
19 40	1	369	9·03	2	0	2
20 12	5	919	8·00	1	0	1
8 plates		2534		10	2	8

The proper motions between 15″ and 20″ and those above 20″ a century are slightly greater in number than the totals for the other four zones.

5. If we arrange the data in Table I.—excluding the supplementary part—in the order of number of stars on a plate instead of in the order of R.A., we should get a general idea of the distribution of the large P.Ms. with reference to the Galaxy, as in Table II. :—

TABLE II.

Plates in order of Number of Stars.

No. Stars.	P.Ms. 15″–20″.	P.Ms. >20″.	No. Stars.	P.Ms. 15″–20″	P.Ms. >20″.	No. Stars.	P.Ms. 15″–20″.	P.Ms. >20″.	No. Stars.	P.Ms. 15″–20″.	P.Ms. >20″.
752	11	3	207	1	1	108	1	0	76	1	1
585	2	1	207	3	1	107	2	0	75	1	0
533	7	1	206	1	0	106	1	2	75	0	2
504	5	1	197	0	3	106	0	1	75	0	1
420	3	0	195	0	0	101	2	0	74	1	0
382	0	2	184	3	0	95	1	0	73	3	3
353	3	1	175	2	2	93	0	1	73	4	1
346	0	2	166	0	1	91	1	1	72	4	0
337	2	1	165	0	1	91	0	2	72	2	3
310	3	1	164	0	1	84	3	1	72	1	1
301	2	1	159	3	1	83	0	2	71	1	1
296	0	0	152	4	3	82	0	2	70	0	1
271	1	0	151	0	0	82	0	0	70	1	0
259	0	1	137	0	0	80	1	1	69	1	0
256	1	3	132	1	1	80	3	1	68	1	1
251	0	0	129	3	1	79	0	2	68	3	0
251	2	1	129	2	3	78	0	0	65	0	0
243	1	1	125	0	1	77	1	0	63	0	0
221	0	2	113	0	0	76	1	0	63	0	0
215	5	1	109	1	1	76	0	3	61	0	0
									61	1	1
									49	0	2
Sums of Groups—											
4522	36	13	1866	10	10	982	11	8	737	17	12
2564	12	10	1336	14	11	793	6	11	778	8	6

These results appear to show that the number of large P.Ms. for a plate in the Galaxy does not greatly exceed that for plates furthest away from the Galaxy, as mentioned in the Second and Third Notes. But if we represent these numbers by the average number of stars in each group for each P.M. star above 15″, we should obtain the following numbers respectively, 9, 12, 9, 5, 5, 5, 3, 5.

6. We may also appropriately arrange the plates in groups in the order of galactic latitude; the results are in Table III.

TABLE III.

Arrangement in Galactic Latitude.

No. of Plates.	Mean Gal. Lat.	Mean Number of P.Ms. per Plate.			Mean No. of Stars.
		All.	15″ to 20″.	>20″.	
16	4°·9	5·9	3·0	1·4	315
15	21·2	5·7	1·3	0·9	247
17	37·3	4·6	0·6	0·9	128
14	58·7	5·1	1·5	1·1	85
20	78·7	4·8	0·8	0·7	74

In this table there are clear indications that the P.Ms. greater than 15″ a century are more numerous in the Galaxy than near the galactic pole: if we take the figures which include all stars this feature is less obvious. But what is evident is a distinct drop in the numbers in the first and second groups for 37° galactic latitude and a rise in all three groups for 59° galactic latitude; this seems something more than being merely accidental; it appears in the results in the Third Note, Table III. (*M.N.*, lxxi. p. 47).

Attention may be directed to the fact that in zone +25° we have, from the 82 plates compared, 1·19 stars per square degree with a proper motion exceeding 10″ a century with more than 9·74 years' interval; between 15″ and 20″ there is 0·32 star per square degree, or one star to each 3·1 square degree; and with proper motions greater than 20″ there is 0·23 star per square degree, or one to each 4·5 square degrees. In other words, if the area of each plate be taken to equal four and a third square degrees, there are 5·2, 1·4, and 1·0 stars per plate for the three groups. These results are slightly larger than in the other zones.

7. The tabulation of the details for each P.M. may be found in Tables IV. and V. The first column is the R.A. of the plate centre for 1900·0; the second column is the approximate galactic latitude; the third and fourth columns give the star's number and the photographic magnitude,† as deduced by the formula given for each plate in vol. vii. of the Oxford Astrographic Catalogue; the fifth column is the number in the Cambridge or Berlin A.G. Catalogues; the sixth column contains the centennial P.M.; the seventh and eighth columns give the Δx and Δy in *réseau* intervals (unit ·001 = 0″·3) between the pairs of plates for the interval in the ninth column.

The sign * in the third column shows that the star is in the A.G. Catalogues.

The pole of the Galaxy has been taken as R.A. $12^h\ 41^m$ +27° 21′.

† These photographic magnitudes of course differ from visual magnitudes, sometimes considerably, *e.g.* Nos. 25°, 45401, 45706, 84556, and 85820 have photographic magnitudes 8·3, 7·6, 8·5, and 8·3, but the B.D. magnitudes are 9·3, 9·3, 9·4, and 9·4. Further, No. 84503, mag. 8·5 photographic, is not in B.D.

TABLE IV.

List of P.Ms. in Zone +25°.

R.A. of Plate Centre.	Approx. Galac. Lat.	Oxford. No. in +25°.	Oxford. Mag. (Photo.)	A.G.C. No.	Cent. P.M. in Arc.	Δx.	Δy.	Interval in Years.
h m					″	R.I.	R.I.	
14 20	68°	42915*	9·6	B 5073	141·9§	+·032	−·040	10·84
14 20	68	42907*	9·6	B 5072	133·4§	+ 28	− 39	10·84
15 0	59	44446*	9·8	C 7086	99·5‡	− 28	+ 16	9·74
16 4	45	46692*	6·4	C 7468	92·2	− 27	+ 40	15·70
19 24	3	61186*	6·0	B 7012	69·7	− 8	− 27	12·13
21 40	21	74584*	9·1	B 8366	68·3	− 15	− 20	10·95
15 16	55	45126*	7·6	C 7155	56·5	− 29	− 6	15·71
13 56	72	86067	10·4	...	53·6	− 30	− 4	16·90
16 52	34	48511	9·9	...	52·1	− 4	− 25	16·90
14 20	68	42937	10·3	...	49·9	− 18	+ 1	10·84
19 32	2	62011	11·6	...	48·6	+ 1	− 18	11·13
18 20	15	55015	11·0	...	45·9	− 3	− 23	15·15
1 16	36	4674	11·0	...	45·7	+ 22	− 10	15·87
15 8	57	44634	9·3	...	44·1	− 22	+ 7	15·70
6 12	6	21321*	9·8	C 3090	42·4	+ 3	− 21	14·97
2 4	34	6725	12·4	...	41·8	+ 23	− 5	16·84
11 0	66	38751*	7·4	C 5620	41·6	− 22	− 1	15·88
10 4	54	36533*	7·9	B 3953	41·1	− 22	+ 7	16·88
12 28	86	40829	10·5	...	39·8	− 22	− 2	16·63
8 4	28	32087*	6·4	C 4397	36·5	− 7	− 18	15·96
3 32	24	10237*	7·6	C 1758	35·4	+ 13	− 15	17·04
6 36	11	24628	9·9	...	34·9	+ 1	− 7	6·10
7 0	15	26801	10·4	...	34·9	− 5	− 5	6·10
16 28	40	47401	10·1	...	33·9	− 3	− 18	16·16
17 8	30	49400*	8·9	B 5874	33·0	− 8	+ 9	10·94
19 8	5	59684	10·1	...	32·7	+ 10	+ 9	12·41
20 20	6	67928	10·0	...	32·0	+ 4	− 10	10·14
14 4	70	42516	11·6	...	31·0	− 16	− 3	15·79
8 4	28	31761*	9·3	C 4379	30·4	− 2	− 16	15·96
12 44	85	41380*	6·2	C 6253	30·4	− 16	− 6	16·89
19 40	1	62821	10·2	...	30·2	− 1	+ 9	9·03
6 36	11	24230*	8·3	B 2526	30·0	+ 1	− 6	6·10
6 20	8	21917	9·9	...	29·8	+ 1	− 10	10·17
4 52	9	13015*	8·5	B 1602	29·7	+ 5	− 13	14·02
16 28	40	47566*	6·6	C 7703	29·4	+ 15	− 5	16·16

§ See *M.N.*, lx. p. 125.

‡ *M.N.*, lvi. p. 9.

TABLE IV.—*continued.*

R.A. of Plate Centre.	Approx. Galac. Lat.	Oxford. No. in +25°.	Mag. (Photo.)	A.G.C. No.	Cent. P.M. in Arc.	Δx. R.I.	Δy. R.I.	Interval in Years.
h m					″			
13 16	81°	84038*	8·8	C 6430	29·2	− 16	+ 4	16·90
19 16	4	60520*	8·4	C 9797	27·8	+ 5	+ 10	12·11
1 32	36	5358*	7·0	B 490	27·7	− 11	− 10	16·16
3 16	26	9303	10·9	...	27·7	+ 9	− 13	17·01
9 56	52	36397*	7·3	C 5179	27·6	− 11	− 1	11·95
19 40	1	62935*	6·8	B 7229	26·9	+ 4	− 7	9·03
7 0	15	26742	11·3	...	26·6	− 2	− 5	6·10
7 0	15	27040	11·3	...	26·6	+ 2	− 5	6·10
13 0	83	83351*	8·5	B 4698	26·6	− 13	+ 7	16·63
13 0	83	83352*	8·8	B 4699	26·6	− 13	+ 7	16·63
13 24	79	84215*	7·8	B 4783	26·4	− 10	+ 11	16·89
14 28	66	43368*	7·8	C 6848	25·7	− 10	+ 9	15·80
12 44	85	41275*	6·7	C 6241	25·6	− 8	− 12	16·89
5 56	2	19431	10·3	...	25·4	− 3	− 8	10·05
15 40	50	45706	7·6	...	25·4	+ 8	− 2	9·74
22 4	24	76243	10·3	...	25·4	+ 10	− 2	12·03
18 52	8	58774	10·8	...	25·2	− 3	− 10	12·41
12 20	84	40724*	7·0	C 6079	25·0	− 13	+ 5	16·63
13 24	79	84322*	6·8	C 6514	24·8	+ 1	− 14	16·89
22 28	28	78791	10·2	...	24·7	+ 4	− 9	11·93
16 52	34	48484	10·2	...	24·2	+ 8	− 11	16·90
3 16	26	9454	12·3	...	23·9	+ 11	+ 8	17·01
14 36	64	43633	10·5	...	23·8	+ 6	+ 11	15·79
10 12	56	37228	10·7	...	23·7	− 7	− 5	10·86
15 8	57	44789*	9·3	C 7136	23·7	− 12	− 3	15·70
22 4	24	76632*	6·8	C 13283	23·4	− 5	− 8	12·03
23 24	34	81611*	9·7	B 9025	23·4	+ 6	− 6	10·87
9 32	47	35684*	9·5	C 5010	23·2	− 9	− 2	11·90
22 28	28	78766	10·2	...	23·2	+ 6	− 7	11·93
5 8	7	13774	10·4	...	23·1	+ 2	− 12	15·85
1 16	36	4601	11·2	...	23·0	2	− 12	15·87
20 12	5	66476	11·3	...	22·9	− 1	− 6	8·00
5 24	4	15048	12·1	...	22·8	− 3	− 7	9·99
5 40	1	16595	11·1	...	22·8	+ 1	− 12	15·78
5 32	3	15921	11·8	...	22·7	− 3	− 7	10·50
5 56	2	19389	12·1	..	22·7	− 3	− 7	10·05
13 56	72	85845	9·6	...	22·3	+ 4	− 12	16·90
2 28	31	7704	11·6	...	22·2	+ 12	− 3	16·80

TABLE IV.—*continued.*

R.A. of Plate Centre.	Approx. Galac. Lat.	Oxford. No. in +25°.	Mag. (Photo.)	A.G.C. No.	Cent. P.M. in Arc.	Δx. R.I.	Δy. R.I.	Interval in Years.
h m					″			
4 52	9°	12907	10·7	...	21·9	− 5	− 9	14·02
4 52	9	13190*	8·5	C 2242	21·6	+ 1	+ 10	14·02
5 16	6	14711	10·6	...	21·6	+ 6	− 6	11·82
6 4	4	20593	11·3	...	21·4	+ 5	− 5	9·97
1 32	36	5303	10·6	...	21·2	+ 9	− 7	16·16
5 56	2	18018	10·7	...	21·2	− 1	− 7	10·05
13 48	74	85659	11·0	...	21·2	− 12	+ 1	16·90
10 12	56	36905*	5·9	B 3971	21·0	− 7	+ 3	10·86
6 20	8	22344*	9·0	C 3261	20·9	− 5	− 5	10·17
22 4	24	76440*	3·2	B 8525	20·4	+ 8	− 2	12·03
16 52	34	48483	10·4	...	20·3	+ 7	− 9	16·90
22 28	28	78897	10·7	...	20·3	+ 1	− 8	11·93
6 12	6	20958	11·0	...	20·2	− 1	− 10	14·97
9 40	49	35934	11·1	...	20·2	− 6	− 3	9·92
10 12	56	37146	11·6	...	20·1	+ 2	− 7	10·86
12 44	85	41423	9·6	...	20·1	− 8	− 8	16·89
21 48	23	75309*	8·5	C 13020	19·9	+ 7	− 2	10·97
19 16	4	60288	10·3	...	19·8	0	− 8	12·11
21 40	21	74970*	8·8	C 12834	19·7	+ 4	+ 6	10·95
21 40	21	74979	9·7	...	19·7	+ 4	+ 6	10·95
19 32	2	61930	11·4	...	19·4	+ 4	− 6	11·13
11 24	71	39233*	9·3	C 5764	19·3	− 6	− 2	9·81
0 52	37	3364	10·9	...	19·2	+ 10	− 2	15·95
13 32	78	84520*	9·0	B 4838	19·2	− 6	− 9	16·89
17 16	28	50157	11·1	...	19·2	− 10	+ 4	16·85
1 16	36	4782	9·8	...	19·0	+ 10	+ 1	15·87
0 52	37	3283*	8·0	B 293	18·8	+ 10	0	15·95
15 0	59	44437*	4·7	C 7084	18·8‡	+ 6	− 1	9·74
15 40	50	45920	9·0	...	18·8	+ 6	+ 1	9·74
6 20	8	22337	10·9	...	18·6	+ 2	− 6	10·17
1 24	36	5194	9·8	C 799	18·4	− 7	− 6	15·03
4 52	9	13201	10·7	...	18·4	− 7	+ 5	14·02
6 4	4	20674	10·3	...	18·4	− 6	· 1	9·97
9 40	49	35902	11·9	...	18·4	− 1	- 6	9·92
9 40	49	35974	11·3	...	18·4	+ 6	· 1	9·92
13 48	74	85595*	9·0	C 6651	18·4	− 10	+ 3	16·90
18 52	8	58962	10·5	...	18·4	− 7	− 3	12·41

‡ See *M.N.*, lvi. p. 9.

TABLE IV.—*continued.*

R.A. of Plate Centre.	Approx. Galac. Lat.	Oxford. No. in +25°.	Mag. (Photo.)	A.G.C. No.	Cent. P.M. in Arc.	Δx.		Δy.		Interval in Years.
h m					″	R.		R.I.		
21 40	21°	74566	9·2	...	18·4	−	3	−	6	10·95
13 32	78	84556	8·5	...	18·3	+	10	−	2	16·89
17 16	28	50274*	7·2	C 8165	18·3	+	5	−	9	16·85
5 32	3	15677	11·8	...	18·2	−	6	−	1	10·05
5 32	3	15974	12·0	...	18·2	−	1	−	6	10·05
21 0	15	71424	10·1	...	18·2	−	3	−	6	11·03
9 16	45	35212*	6·6	C 4921	18·1	+	4	−	6	11·92
13 48	74	85663*	9·1	C 6659	18·1	+	2	−	10	16·90
6 20	8	22409	10·9	...	18·0	+	1	−	6	10·17
14 44	62	43742*	5·7	B 5182	18·0	+	9	+	3	15·79
5 40	1	16111	11·7	...	17·9	−	5	−	8	15·78
13 32	78	84742	9·6	...	17·9	−	10	+	1	16·89
13 8	82	83611	10·0	...	17·7	−	10		0	16·90
14 20	68	42946	9·6	...	17·7	+	4	−	5	10·84
6 4	4	20562	11·5	...	17·5	−	5	−	3	9·97
9 40	49	35924*	2·1	B 3836	17·5	+	3	+	5	9·92
13 48	74	85498	10·0	...	17·5	−	9	+	4	16·90
19 24	3	61093	10·3	...	17·5	−	1	−	7	12·13
5 24	4	14904*	8·0	C 2460	17·4	+	3	+	5	9·99
10 12	56	36908	11·4	...	17·4	−	2	−	6	10·86
18 52	8	58767	10·2	...	17·4	−	4	−	6	12·41
21 0	15	72018*	9·1	C 12058	17·4	+	4	+	5	11·03
22 4	24	76666	10·6	...	17·4		0	−	7	12·03
5 32	3	15850*	4·6	C 2575	17·3	−	5	+	3	10·05
5 40	1	16973*	10·3	C 2681	17·3	+	1	−	9	15·78
13 16	81	83999	9·8	...	17·3	−	9	+	4	16·90
19 8	5	59674	11·1	...	17·2	−	7	+	1	12·41
20 20	6	67681	10·3	...	17·2	+	3	+	5	10·14
5 24	4	15034	12·1	...	17·1	−	4	−	4	9·99
5 56	2	18005	11·4	...	17·0	−	4	−	4	10·05
11 0	66	38730	9·7	...	17·0	+	9		0	15·88
14 20	68	43168	10·0	...	16·9	+	1	−	6	10·84
22 28	28	79180	11·5	...	16·9	+	3	+	6	11·93
10 12	56	37173	10·6	...	16·8	+	6	+	1	10·86
10 52	65	38527	11·7	...	16·8	+	3	−	4	8·93
10 52	65	38555*	8·7	C 5565	16·8	+	5		0	8·93
18 20	15	54820	9·9	...	16·8	+	3	−	8	15·15
14 36	64	43645*	6·9	C 6900	16·7	+	7	−	6	15·79

TABLE IV.—*continued.*

R.A. of Plate Centre.	Approx. Galac. Lat.	Oxford. No. in +25°.	Mag. (Photo.)	A.G.C. No.	Cent. P.M. in Arc.	Δx. R.I.	Δy. R.I.	Interval in Years.
h m					″			
21 40	21°	74981*	7·5	C 12922	16·7	+ 1	+ 6	10·95
13 40	76	85098	10·7	...	16·6	− 8	+ 5	16·90
11 24	71	39280	9·8	...	16·5	+ 5	− 2	9·81
11 40	75	39652	10·8	...	16·5	− 5	+ 2	9·81
10 28	60	37904	8·3	C 5421	16·4	− 9	− 2	16·88
17 16	28	50281	10·9	...	16·4	− 7	− 6	16·85
5 16	6	14709	10·9	...	16·3	− 4	+ 5	11·82
6 4	4	19576	10·8	...	16·3	+ 5	+ 2	9·97
6 4	4	19731	10·1	...	16·3	+ 5	− 2	9·97
14 44	62	43716	9·4	...	16·3	− 7	+ 5	15·79
15 16	55	45116*	8·7	C 7169	16·3	− 3	− 8	15·71
17 0	32	48835	9·7	...	16·3	− 9	− 2	16·92
13 32	78	84720	10·8	...	16·2	− 1	− 9	16·89
14 4	70	42502	9·4	...	16·2	− 3	− 8	15·79
19 32	2	62143	11·6	...	16·2	+ 6	0	11·13
5 56	2	18298*	7·5	B 2077	16·1	+ 2	+ 5	10·05
5 56	2	18371	11·6	...	16·1	− 2	+ 5	10·05
5 56	2	18526	12·3	...	16·1	+ 2	+ 5	10·05
5 32	3	15319	11·8	...	16·1	− 5	− 2	10·05
11 0	66	38642*	8·4	C 5593	16·1	+ 3	− 8	15·88
12 12	82	40411	11·3	...	16·1	+ 4	− 4	10·63
14 44	62	43795*	6·1	C 6976	16·1	− 6	+ 6	15·79
20 20	6	68559*	10·2	C 11281	16·0	− 5	− 2	10·14
9 32	47	35610	10·9	...	15·9	− 2	+ 6	11·90
21 40	21	74627	11·1	...	15·9	− 3	− 5	10·95
2 4	34	6810	13·4	...	15·8	− 4	+ 8	16·84
15 0	59	44307	10·0	...	15·7	− 1	− 5	9·74
15 0	59	44316*	8·5	B 5215	15·7	+ 5	− 1	9·74
19 24	3	61246	10·8	...	15·6	− 2	− 6	12·13
5 8	7	14100*	8·1	C 2330	15·5	− 2	− 8	15·85
18 52	8	58883	9·3	...	15·5	+ 4	+ 5	12·41
19 8	5	59667*	8·6	B 6925	15·5	− 5	− 4	12·41
21 0	15	71511	9·6	...	15·5	+ 4	+ 4	11·03
9 56	52	36494*	10·3	C 5195	15·3	+ 1	+ 6	11·95
11 0	66	38741	11·4	...	15·3	− 8	− 1	15·88
11 0	66	38755	10·2	...	15·3	− 1	− 8	15·88
22 28	28	78681	10·3	...	15·3	− 1	+ 6	11·93
22 28	28	78713	10·3	...	15·3	+ 6	− 1	11·93

TABLE IV.—*continued.*

R.A. of Plate Centre.	Approx. Galac. Lat.	Oxford. No. in +25°.	Mag. (Photo.)	A.G.C. No.	Cent. P.M. in Arc.	Δx.	R.I.	Δy.	R.I.	Interval in Years.
h m					″					
22 28	28°	79000	10·3	...	15·3	+	6	+	1	11·93
4 52	9	13057	11·1	...	15·2	+	1	+	7	14·02
5 56	2	18624	12·5	...	15·2	−	5	−	1	10·05
5 56	2	18769*	7·1	C 2846	15·2	+	1	+	5	10·05
5 56	2	19314	11·7	...	15·2	−	1	−	5	10·05
5 56	2	18003	10·9	...	15·2	−	5	−	1	10·05
6 4	4	20675	11·0	...	15·1	-	5		0	9·97
19 24	3	61073*	5·3	B 7046	15·1	−	6	−	1	12·13
20 20	6	67645·	9·3	...	15·1	−	1	+	5	10·14
3 24	25	9609	11·7	...	15·0	+	6	−	6	17·04
5 24	4	15056	11·5	...	15·0	−	3	−	4	9·99
5 32	3	15119	12·0	...	15·0	+	5		0	10·05
5 32	3	15865	12·4	...	15·0	−	3	−	4	10·05
5 32	3	15902	12·0	...	15·0		0	−	5	10·05
5 56	2	18244*	9·2	B 2080	15·0	+	4	+	3	10·05
5 56	2	19283*	7·5	C 2925	15·0		0	+	5	10·05
5 56	2	19399	11·7	...	15·0	−	4	−	3	10·05
14 20	68	43180*	5·7	C 6814	15·0	−	5	+	2	10·84
18 20	15	55553*	9·5	C 8890	15·0	−	3	−	7	15·15
1 32	36	5404*	5·6	*C 931	14·9	+	8		0	16·16
10 12	56	36911	10·2	...	14·9	−	2	−	5	10·86
23 8	33	80845*	9·6	B 8876	14·9	+	5	−	2	10·87
12 20	84	40637*	6·7	B 4514	14·8	+	8	+	2	16·63
19 24	3	61731*	8·3	C 10004	14·8		0	+	6	12·13
21 0	15	71686	11·2	...	14·7	−	2	−	5	11·03
21 0	15	71871	9·8	...	14·7	−	2	−	5	11·03
2 4	34	6917	11·0	...	14·6	+	8	−	2	16·84
12 20	84	40650	10·8	...	14·6	−	4	+	7	16·63
13 32	78	84506	9·8	...	14·6	+	2	−	8	16·89
16 4	45	46714	10·2	...	14·6	+	3	−	7	15·70
17 0	32	49047*	8·2	C 7969	14·6	−	2	−	8	16·92
2 4	34	6759*	9·3	C 1181	14·4	−	4	−	7	16·84
5 8	7	14340*	8·8	C 2334	14·4	+	3	−	7	15·85
10 4	54	36683*	8·5	C 5229	14·4	−	7	−	4	16·88
14 36	64	43522*	8·7	B 5130	14·4	−	7	+	3	15·79
19 16	4	60166*	7·7	B 6940	14·4	+	3	+	5	12·11
22 4	24	76216	9·9	...	14·4	+	5	+	3	12·03
22 28	28	78979*	8·2	C 13492	14·3	+	4	−	4	11·93

TABLE IV.—*continued.*

R.A. of Plate Centre.	Approx. Galac. Lat.	Oxford. No. in +25°.	Mag. (Photo.)	A.G.C. No.	Cent. P.M. in Arc.	Δx.		Δy.		Interval in Years.
h m					"	R.I.		R.I.		
6 12	6°	21337	11·8	...	14·2	+	5	−	5	14·97
16 28	40	47451*	9·3	C 7664	14·2	−	7	+	3	16·16
10 12	56	37237	8·9	..	14·1	+	5	−	1	10·86
14 20	68	43062	10·3	..	14·1	−	1	+	5	10·84
23 8	33	80853*	6·5	B 8904	14·1	+	5	−	1	10·87
23 8	33	81059	11·0	...	14·1	+	5	+	1	10·87
3 16	26	9516	11·7	...	14·0	+	8		0	17·01
13 0	83	83367	11·7	...	14·0	+	6	+	5	16·63
13 32	78	84592	10·2	...	13·9	−	6	+	5	16·89
1 16	36	4744*	9·2	C 735	13·8	+	7	−	2	15·87
15 32	52	45548*	8·1	C 7256	13·8	+	6	−	4	15·71
19 8	5	59501	9·7	...	13·8	+	4	+	4	12·41
1 16	36	4659*	7·4	B 405	13·6	+	6	−	4	15·87
11 0	66	38625	10·9	...	13·6	−	4	+	6	15·88
13 8	82	83640*	7·4	B 4707	13·6	−	7	+	3	16·90
17 48	24	52260*	9·7	C 8455	13·6	+	3	+	7	16·73
21 0	15	72293	9·6	...	13·6		0	−	5	11·03
22 28	28	78899	11·4	...	13·6	−	2	−	5	11·93
22 28	28	78918	9·2	...	13·6	−	2	−	5	11·93
2 4	34	6978	10·9	...	13·5	−	3	−	7	16·84
12 36	86	41066	10·2	...	13·5	+	6	−	4	15·94
14 36	64	43504	10·9	...	13·5	−	1	−	7	15·79
19 32	2	62335	11·6	...	13·5	−	5		0	11·13
8 4	28	31820*	9·2	C 4398	13·4	+	1	−	7	15·96
14 44	62	43862	10·6	...	13·4	−	1	+	7	15·79
1 48	35	5922	9·8	...	13·2	+	1	−	7	16·16
1 48	35	5971*	6·9	C 1028	13·2	+	7	+	1	16·16
1 48	35	6020*	9·1	C 1074	13·2	−	5	−	5	16·16
8 4	28	32126	10·5	...	13·2	−	7		0	15·96
12 4	80	40205	10·6	...	13·2		0	−	7	15·94
13 0	83	83303*	8·7	B 4702	13·1	−	7	−	2	16·63
13 0	83	83502	10·2	...	13·1	−	7	+	2	16·63
1 48	35	5907	10·5	...	13·0		0	−	7	16·16
13 32	78	84598	10·8	...	13·0	+	7	+	2	16·89
13 32	78	84762	10·8	...	13·0	+	7		2	16·89
16 52	34	48250	9·9	...	13·0	−	7	−	2	16·90
18 36	12	56900	9·3	...	13·0	−	7		0	16·14
9 16	45	35063	10·9	...	12·9	−	5	+	1	11·92

TABLE IV.—*continued.*

R.A. of Plate Centre.	Approx. Galac. Lat.	Oxford. No. in +25°.	Mag. (Photo.)	A.G.C. No.	Cent. P.M. in Arc.	Δx. R.I.		Δy. R.I.		Interval in Years.
h m					"					
3 8	27°	9103	12·9	...	12·8	+	6	−	4	17·11
4 52	9	12918*	7·6	B 1593	12·8	−	6		0	14·02
12 28	86	40847	9·8	...	12·8	+	5	−	5	16·63
12 28	86	40891	9·2	...	12·8	+	1	−	7	16·63
15 32	52	45494*	10·4	C 7260	12·8	+	3	+	6	15·71
17 0	32	49003*	6·5	C 7959	12·8	+	4	+	6	16·92
22 28	28	78951	10·3	...	12·8	−	1	−	5	11·93
1 16	36	4665	12·3	...	12·7	+	3	−	6	15·87
5 16	6	14429	9·3	...	12·7	+	3	−	4	11 82
8 4	28	32076	9·6	...	12·7	−	3	−	6	15·96
12 4	80	40231	10·7	...	12·7	+	3	−	6	15·94
2 4	34	6605	12·8	...	12·6	+	1	−	7	16·84
2 4	34	6673	10·9	...	12·6	+	7	+	1	16·84
3 16	26	9512	11·1	...	12·6	−	6	+	4	17·01
9 16	45	35156	10·7	...	12·6	−	3	+	4	11·92
13 16	81	83938	11·0	...	12·6	−	1	−	7	16·90
13 40	76	85101	10·5	...	12·6	+	7	−	1	16·90
13 48	74	85455	10·6	...	12·6	−	1	−	7	16·90
16 52	34	48245	10·6	...	12·6	+	5	−	5	16·90
19 16	4	60425	10·4	...	12·6	+	1	−	5	12·11
3 24	25	9647	12·3	...	12·5	−	5	+	5	17·04
22 4	24	76536*	9·2	C13177	12·5		0	−	5	12·03
22 4	24	76591	10·3	...	12·5	−	5		0	12·03
13 56	72	85820	8·3	...	12·4		0	−	7	16·90
17 0	32	48975*	9·8	C 8020	12·4		0	−	7	16·92
5 40	1	16477	11·2	...	12·2	−	5	−	4	15·78
6 12	6	21195	11·8	...	12·2	+	6	−	1	14·97
14 28	66	43327*	7·6	B 5110	12·2	−	4	+	5	15·80
15 32	52	45401	8·3	...	12·2	+	4	−	5	15·71
16 4	45	46654	10·5	...	12·2	+	5	−	4	15·70
13 0	83	83333	10·5	...	12·1	−	6	+	3	16·63
15 32	52	45495	11·2	...	12·1	+	2	+	6	15·71
16 4	45	46551	10·0	...	12·1	+	2	+	6	15·70
19 8	5	59655*	9·1	B 6868	12·1		0	−	5	12·41
2 28	31	7603	11·0	...	12·0	−	3	−	6	16·80
14 28	66	43306	10·7	...	12·0	−	6	−	2	15·80
13 32	78	84503	8·5	...	11·9	−	3	−	6	16·89
13 32	78	84653*	8·5	C 6552	11·9	+	3	+	6	16·89

TABLE IV.—*continued.*

R.A. of Plate Centre.	Approx. Galac. Lat.	Oxford. No. in +25°.	Mag. (Photo.)	A.G.C. No.	Cent. P.M. in Arc.	Δx. R.I.	Δy. R.I.	Interval in Years.
h m					"			
13 48	74°	85689*	8·4	C 6624	11·9	− 6	+ 3	16·90
13 56	72	85975	10·6	...	11·9	− 6	− 3	16·90
13 24	79	84391	11·1	...	11·9	+ 6	+ 3	16·89
3 16	26	9342	12·5	...	11·7	+ 6	+ 3	17·01
5 40	1	16944*	7·8	C 2600	11·6	+ 6	+ 1	15·78
14 4	70	42581	10·5	...	11·6	+ 6	− 1	15·78
8 4	28	31520	12·1	...	11·5	− 1	− 6	15·96
12 36	86	41100	10·1	...	11·5	− 6	+ 1	15·94
16 4	45	46705	11·1	...	11·5	0	+ 6	15·70
2 4	34	6851	12·4	...	11·4	+ 5	− 4	16·84
2 4	34	6940	12·8	...	11·4	+ 5	+ 4	16 84
10 28	60	37612*	9·1	B 4064	11·4	+ 4	− 5	16·88
13 32	78	84613	10·0	...	11·4	+ 5	− 4	16·89
0 52	37	3290*	7·1	B 313	11·3	+ 6	0	15·95
8 4	28	31690*	9·0	C 4374	11·3	− 6	0	15·96
12 4	80	40286*	7·3	C 6020	11·3	− 6	0	15·94
12 28	86	40873	10·8	...	11·3	− 2	− 6	16·63
13 8	82	83639	9·1	...	11·3	+ 5	− 4	16·90
13 16	81	83901	10·8	...	11·3	− 5	− 4	16·90
2 4	34	6702	11·9	...	11·2	− 6	+ 2	16·84
2 4	34	6721	13·0	...	11·2	+ 6	− 2	16·84
2 4	34	6825	12·2	...	11·2	− 6	+ 2	16·84
2 4	34	6843	12·6	...	11·2	+ 6	− 2	16·84
17 16	28	50319*	9·0	C 8114	11·2	− 2	− 6	16·85
17 16	28	50621*	7·7	C 8152	11·2	+ 2	+ 6	16·85
5 40	1	16034	10·7	...	11·0	+ 3	+ 5	15·78
8 4	28	31683	9·3	...	11·0	− 5	− 3	15·96
11 0	66	38691	9·7	...	11·0	− 5	+ 3	15·88
12 28	86	40801	11·1	...	11·0	+ 1	− 6	16·63
2 4	34	6806	11·9	...	10·9	+ 6	+ 1	16·84
10 28	60	37911	10·4	...	10·9	− 6	+ 1	16·88
13 40	76	85022	9·1	...	10·8	+ 6	+ 1	16·90
17 0	32	49079	10·3	...	10·8	+ 1	− 6	16·92
18 36	12	57346*	9·7	C 9103	10·8	− 3	− 5	16·14
2 28	31	7720	13·0	...	10·7	− 6	0	16·80
3 24	25	9868	12·1	...	10·7	− 6	− 1	17·04
4 52	9	12927	10·4	...	10·7	+ 5	0	14·02
10 28	60	37803	10·5	...	10·7	0	− 6	16·88
11 56	79	40009	11·0	...	10·7	− 4	− 4	16·00

TABLE IV.—*continued.*

R.A. of Plate Centre.	Approx. Galac. Lat.	Oxford. No. in +25°.	Mag. (Photo.)	A.G.C. No.	Cent. P.M. in Arc.	Δx.		Δy.		Interval in Years.
h m					″	R.I.		R.I.		
17 16	28°	50318	10·8	...	10·7		0	–	6	16·85
3 24	25	9629	9·6	...	10·6	–	6		0	17·04
13 8	82	83737	10·8	...	10·6	–	6		0	16·90
13 40	76	85291*	8·8	C 6582	10·6	–	6		0	16·90
13 24	79	84388	10·1	...	10·6		0	–	6	16·89
17 0	32	48709	10·3	...	10·6		0	+	6	16·92
17 0	32	48769	10·2	...	10·6		0	-	6	16·92
17 0	32	48783*	7·7	B 5859	10·6		0	+	6	16·92
12 20	84	40659	9·7	...	10·4	–	5	+	3	16·63
16 52	34	48028	10·3	..	10·4	+	3	–	5	16·90
17 16	28	50143	10·5	...	10·4	–	3	+	5	16·85
5 40	1	16257	11·5	...	10·3	–	2	–	5	15·78
5 40	1	16747	12·2	...	10·3	-	5	–	2	15·78
12 44	85	41449*	10·0	C 6275	10·3		5	–	3	16·89
12 44	85	41457	10·0	...	10·3	+	3	–	5	16·89
13 8	82	83758*	9·7	C 6387	10·3	+	3	-	5	16·90
13 16	81	83966	11·7	...	10·3	+	5	–	3	16·90
13 16	81	84014	11·4	...	10·3	+	5	+	3	16·90
13 24	79	84222	10·3	...	10·3	–	5	–	3	16·89
14 36	64	43618	10·1	...	10·3	–	2	+	5	15·79
3 16	26	9363	11·9	...	10·2	+	5	–	3	17·01
3 16	26	9497	12·5	...	10·2	–	3	–	5	17·01
3 16	26	9548	11·1	...	10·2	–	5	–	3	17·01
5 8	7	13726	11·2	...	10·2	–	2	–	5	15·85
6 12	6	20862	9·6	...	10·2	–	1	–	5	14·97
6 12	6	21284	10·1	...	10·2	+	5	–	1	14·97
12 4	80	40247*	10·3	C 6006	10·2	+	5	+	2	15·94
18 12	18	54105*	7·0	B 6349	10·2	+	4	+	4	16·76
10 4	54	36554	8·3	...	10·1	–	4	–	4	16·88
18 20	15	54803*	7·9	B 6426	10·1	-	1	–	5	15·15
18 20	15	55538*	10·0	C 8852	10·1	–	1	–	5	15·15
1 48	35	6017	12·0	...	10·0	+	5	+	2	16·16
3 24	25	9749	12·9	...	10·0	+	4	–	4	17·04
6 12	6	20990	10·1	...	10·0	–	4	+	3	14·97
6 12	6	21510	10·6	...	10·0	+	3	–	4	14·97
12 44	85	41307	10·7	...	10·0	+	4	+	4	16·89
12 44	85	41372	10·0	...	10·0	+	4	+	4	16·89
16 28	40	47432	9·3	...	10·0	+	2	+	5	16·16
18 36	12	56738*	7·5	B 6570	10·0	–	2	–	5	16·14

8. On account of the greater interval in years the following fifty-five stars with P.Ms. below 10″ seem worthy of record, quite as much as some of the stars above 10″ given in Table IV. and in the other zones when the interval is much shorter.

In the comparison of these pairs of plates for proper motions any uncertainty of the plate constants, which are based upon the A.G. Catalogues, will affect the values of the centennial P.Ms. given in Tables IV. and V.

TABLE V.

Fifty-five Stars with P.Ms. below 10″ a century.

R.A. of Plate Centre.	Approx. Galac. Lat.	Oxford-No. in +25°.	Mag. (Photo.)	A.G.C. No.	Cent. P.M. in Arc.	Δα.	Δδ.	Interval in Years.
h m.					″	R.I.	R.I.	
15 8	57°	44823	9·7	...	9·7	- 1	5	15·70
15 16	55	44927	9·9	...	9·7	- 5	1	15·71
15 16	55	44957	10·6	...	9·7	- 5	- 1	15·71
3 32	24	10074	10·1	...	9·6	+ 2	- 5	17·04
3 32	24	10274	10·8	...	9·6	- 2	5	17·04
8 4	28	31786	11·7	...	9·6	+ 5	1	15·96
10 28	60	37650	9·3	...	9·6	5	2	16·88
11 56	79	40066*	9·0	C 5931	9·6	5	- 1	16·00
12 44	85	41243	9·7	...	9·6	5	+ 2	16·89
12 44	85	41366*	9·2	C 6251	9·6	+ 5	- 2	16·89
13 16	81	84097	11·7	...	9·6	+ 5	+ 2	16·90
13 56	72	85967	10·6	...	9·6	- 2	- 5	16·90
15 8	57	44759	10·6	...	9·6	- 3	+ 4	15·70
16 52	34	48021*	8·2	B 5761	9·6	- 5	+ 2	16·90
16 52	34	48469	10·3	...	9·6	5	2	16·90
17 0	32	49030	10·1	...	9·6	2	- 5	16·92
17 48	24	52546	9·6	...	9·6	+ 2	+ 5	16·73
3 8	27	9152	12·5	...	9·5	- 5	+ 2	17·11
3 16	26	9551*	9·7	C 1633	9·5	5	2	17·01
5 40	1	17213*	10·7	C 2702	9·5	0	- 5	15·78
5 40	1	17044	11·1	...	9·5	0	5	15·78
8 4	28	31627*	8·5	B 3260	9·5	- 3	- 4	15·96
11 0	66	38626	11·5	..	9·5	- 4	+ 3	15·88
11 0	66	38718*	10·5	C 5626	9·5	- 5	0	15·88
12 4	80	40248*	8·3	C 6010	9·5	0	+ 5	15·94
15 16	55	44924	9·3	...	9·5	0	+ 5	15·71
15 16	55	45059*	9·9	C 7149	9·5	5	0	15·71
12 36	86	41003	10·9	...	9·4	0	- 5	15·94
1 48	35	5964*	8·4	C 993	9·3	- 5	0	16·16
12 28	86	40893*	4·9	C 6145	9·2	- 1	+ 5	16·63
13 0	83	83302	9·6	...	9·2	- 1	- 5	16·63

TABLE V.—*continued.*

R.A. of Plate Centre.	Approx. Galac. Lat.	Oxford. No. in +25°.	Mag. (Photo.)	A.G.C. No.	Cent. P.M. in Arc.	Δx. R.I.		Δy. R.I.		Interval in Years.
h m					″					
10 4	54°	36663	9·5	...	9·1	·	5	+	1	16·88
10 4	54	36735	10·7	...	9·1	+	5		1	16·88
12 44	85	41217	10·2	...	9·1	+	1		5	16·89
12 44	85	41226*	8·9	B 4362	9·1		1	−	5	16·89
12 44	85	41454*	8·8	C 6271	9·1		5	−	1	16·89
16 52	34	48101*	7·2	B 5795	9·1	+	1	-	5	16·90
17 16	28	50354*	10·9	C 8124	9·1	+	1		5	16·85
3 24	25	9604	12·1	...	9·0		1		5	17·04
3 24	25	9628*	5·6	B 1057	9·0	−	5	−	1	17·04
3 24	25	9858*	9·6	C 1683	9·0	−	5	−	1	17·04
12 20	84	40644	10·9	...	9·0	+	4	+	3	16·63
12 20	84	40658	11·1	...	9·0	−	5		0	16·63
13 16	72	83955	10·5	...	9·0	+	5	−	1	16·90
13 56	72	85821	8·6	...	9·0	+	1	−	5	16·90
17 0	32	48832*	9·9	C 8033	9·0	-	5	+	1	16·92
18 12	18	54293	10·6	...	9·0	−	4		3	16·76
10 4	54	36646	11·0	...	8·9	+	5		0	16·88
12 44	85	41368	10·6	...	8·9	+	5		0	16·89
13 56	72	85827	10·0	...	8·9		0	−	5	16·90
13 56	72	86084*	8·5	C 6688	8·9		0	+	5	16·90
13 24	79	84277	11·1	...	8·9	−	4	−	3	16·89
17 0	32	48889*	8·5	C 7956	8·9		0	+	5	16·92
17 16	28	50574*	7·3	C 8128	8·9		0	+	5	16·85
3 24	25	9746	12·1	...	8·8	+	5		0	17·04

9. By grouping these 433 proper motions in +25° zone into three-hour groups we get the figures as in Table VI.

TABLE VI.

Distribution of P.Ms. in Three-hour Groups.

R.A.	No. Plates.	No. P.M. Stars.	Mean Difference of Epochs.	Mean P.Ms.
h h			y	″
0– 3	7	36	16·4	15·8
3– 6	11	72	13·7	15·5
6– 9	9	34	12·5	19·2
9–12	14	45	13·5	16·6
12–15	19	111	16·1	17·4
15–18	12	61	15·4	18·8
18–21	11	39	12·5	20·1
21–24	7	35	11·5	18·6
Sum or wt. mean	90	433	14·4	17·8

These figures indicate in a general way that the larger mean P.Ms. correspond to the smaller mean difference between the epochs. The explanation is, of course, that with a longer interval we are able to detect smaller proper motions, which do not show with a shorter interval.

10. If, now, we group these stars according to magnitude, we get results as follow, N denoting the *total* number as bright as, or brighter than, the reading.

TABLE VII.

Showing P.Ms. for successive Magnitudes. +25°.

	To Mag. 7·9.	8·9.	9·9.	10·9.	12·0.
N over 20″	21	29	45	69	89
N 15″ to 20″	14	26	51	85	117
N over 15″	35	55	96	154	206
log. N over 15″	1·54	1·74	1·98	2·19	2·31
Difference		0·20	0·24	0·21	0·12

These limits of magnitudes are photographic and are those given in Table IV., except for the faintest stars, where the formula ceases to give good results. As a rude approximation, the lower limit has been taken as 12·0 for all plates, as in the previous "Notes." The combined results from all the zones +25°, +26°, +27°, +28°, +29° are collected in Table VIII.

TABLE VIII.

Showing P.Ms. for 15″ per century for Successive Magnitudes in Zones +25° to +29°.

N over 15″.	To Mag. 7·9.	8·9.	9·9.	10·9.	12·0.
+25°	35	55	96	154	206
+26	5	14	25	35	52
+27	8	18	27	40	55
+28	3	5	7	11	16
+29	10	18	22	32	63
Total	61	110	177	272	392
Log N over 15″	1·785	2·041	2·248	2·435	2·593
Formula	1·792	2·034	2·248	2·434	2·592
Diff. O—C	−0·007	+0·007	0·000	+0·001	+0·001

The material indicates an approximate value of log N as follows,

$$\log N = 2{\cdot}268 + 0{\cdot}197(m - 10{\cdot}0) - 0{\cdot}014(m - 10{\cdot}0)^2$$

from which the values in the line "Formula" have been calculated. This empirical formula gives a negative rate of increase in log. N after magnitude 17, but may serve to represent the facts approximately to the 14th or 15th magnitude.

University Observatory, Oxford:
1911 November 1.

The Influence of Anomalous Dispersion on Solar Phenomena.
By P. V. Bevan, M.A., Sc.D., Royal Holloway College.

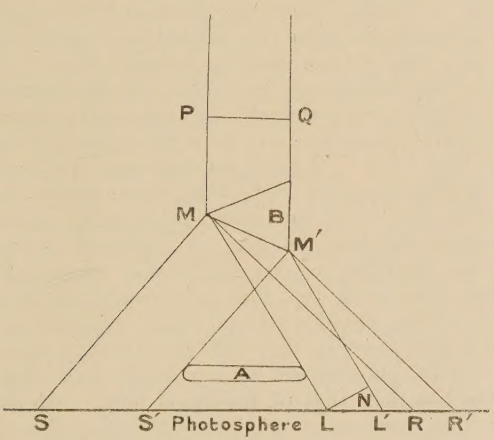
Anomalous dispersion effects with metallic vapours may give rise to apparent reversals of dark absorption lines. The phenomenon occurs when a non-uniform source sends light through a non-homogeneous vapour to a spectroscope slit, if when the vapour is removed from the path of the light the brightest part of the image of the source is just above or just below the slit. I observed the phenomenon in the cases of potassium vapour and lithium in the first instances, and the light that appears in the spectroscope as the reversal of the dark absorption line is with small dispersion indistinguishable from that of the actual line, though it is of wave-length just greater or just less. Julius has made a number of interesting experiments of a similar character, and has discussed their bearing on astrophysical phenomena. Lockyer, in 1874,* observed some remarkable appearances in the absorption spectrum of sodium vapour, in particular flag-like absorption, and apparently unilateral absorption. These phenomena admit of ready explanation by anomalous dispersion with a non-uniform source of light. The theory of Julius is that all wings and differences of lines in absorption spectra are due to anomalous dispersion, but this cannot be maintained in the face of the facts.† An explanation of the bright lines of the chromosphere and the flash spectrum, as due to anomalous dispersion, has been offered, but it seems certain that if this were so the displacement of the lines would be noticeable. The object of the present communication is to show how there may be anomalous dispersion effects in the phenomena of the solar disc, and that, at any rate, some of these may be explained on these lines. Julius assumes ‡ that differences in the solar atmosphere are due to changes in density of a uniform —as regards composition —mixture of gases. These changes are produced by vortices and other disturbances: a tubular structure exists, corresponding to the coronal streamers, and in virtue of density gradients at right angles to the constituent filaments of the structure, effects may be produced similar to those due to the gathering power for light of a bundle of glass tubes. But it is clear that such gathering power is only effective for an observer near to the ends of the tubes. For a distant observer no concentration of light would be noticeable for a uniform large source behind the tubes. A similar structure in the solar atmosphere producing deviation alone, and not absorption, would give no apparent change in the distribution of light from the solar disc.§

For our theory, we assume that broadened lines and wings are really absorption effects, and are not due to dispersion. We shall

* *Proc. Roy. Soc.*, xxii. p. 378.
† See, for example, *Proc. Roy. Soc.*, A, lxxxiii. p. 426, and A, lxxxv. p. 62.
‡ *Astrophysical Journal*, xii. p. 185.
§ See a similar argument by Anderson, *Astrophysical Journal*, xxxi. p. 166.

see that the phenomenon of double reversal can occur on the further assumption that we may have in the solar atmosphere differences of composition of vapour, just as in the Earth's atmosphere we may have masses of air containing large or small quantities of water vapour. This interpretation may therefore explain some of the appearances in spectroheliograms, and some of the phenomena exhibited by such lines as the H and K lines in the solar spectrum. As perhaps most attention has been paid to the appearances involving calcium vapour, we shall take this as an example in the following discussion of the effects which might appear.

For simplicity, consider the photosphere to be a uniformly luminous plane, observed normally from a great distance, and that above it we have a mass of calcium vapour, represented by A in the figure. The spectrum due to light from the region of



the photosphere behind A will have, we suppose, broadened H and K absorption lines due to A. Now suppose that in front of A we have a second mass of calcium vapour. For the present, to simplify the problem, consider this to be prismatic in form, with its refracting edge parallel to the photosphere. Represent this mass of vapour by B. To specify what light may appear to come from B, we must take into account light deviated by the prism that may have come from the photosphere, not directly behind B. B, owing to its greater height from the surface of the Sun, will be, in general, less dense and cooler than A, so that white light passing through B would give narrower H and K lines than those due to A. We may assume that all light that emerges from A normally to the photosphere passes practically unchanged in direction through B, for it is only light of wave-length very near that of the H and K lines that has a refraction index in B appreciably different from 1, and A has already absorbed light of this kind, giving rise to the broadened H and K bands. But

consider light that is emitted from the part of the photosphere LL′ in the direction LM, L′M′. Confining our attention to light of wave-length near one of the H and K lines only, say K, that portion of this light with wave-length near that of K will be appreciably deviated by B. Light of wave-length greater than that corresponding to K will be deviated in the ordinary way for a prism, and for a certain value of the wave-length will emerge from B normally to the photosphere. The intensity of this light, assuming that it is not absorbed in B, will be $\frac{LN}{PQ}$ times the intensity of the direct light of the same wave-length emitted normally by the photosphere. Light of wave-length less than that of this last, but still greater than that of K, can come from a region RR′, and again will appear after transmission through B to be coming normally from the photosphere. We shall have, thus, light of a certain range of wave-lengths emerging from B normally to the photosphere for which the intensity will be not much less than that of the directly emitted light of the same wave-length. The range will be small, for the refraction index of B will be only great enough for light very near to K in wave-length, and in the case we have considered of wave-length greater than that of K. Now if we consider light emitted by a region SS′ in the direction SM, S′M′, that which is of wave-length less than that of K, but nearly equal to it, will be deviated in the opposite direction to that of ordinary refraction, the refractive index for light just on the smaller wave-length side of our absorption line being less than 1. Similar considerations to the above show, then, that for a certain small range of wave-lengths near that of K and less than that of K there will be light travelling normally from the photosphere after passing through B again of intensity little less than that of the direct light. An observer, on the slit of whose spectroscope an image of this region of the Sun falls, would observe then the broadened K line due to absorption by A, and a bright line on each side of it due to light from other parts of the photosphere deviated by B. In other words, we should observe the double reversal. The bright regions designated by Hale H_2 and K_2 for those lines would not, if arising from this cause, be generally symmetrical about the true position of the absorption line. Their breadth and aspect would, of course, depend on the distribution of such masses of vapour as A and B.

In the actual case of the solar atmosphere, if we had a non-homogeneous distribution, masses of vapour would exist which we can regard as built up of prismatic elements, which would behave as the vapour B which we have considered; the same result would follow, and we should expect that, whenever one mass of vapour existed above another, the upper one cooler and not so dense as the lower, double reversals would appear. If we had large numbers of these masses of vapour, provided that they were small compared with the detail visible in a spectroheliogram, we should expect the appearance of such reversals to be fairly permanent.

All kinds of asymmetry would present themselves, and in particular we could have what is apparently a single reversal, with a displaced bright line. We can see also that even if this were the only cause of such double reversal, the reversal might appear in the integrated light of the Sun, for in the case considered there will be, in the normal direction from the whole Sun, more light of wave-length very near to that of the line K than light whose wave-length is further removed, so that the refractive index of B is too small for it to be deviated into the direction normal to the photosphere.

The phenomenon, as thus described, depends on the presence of two regions of absorption—calcium vapour—instead of three on the generally accepted theory, and on the necessity of the upper level vapour being less dense, and so giving sharper absorption lines than the lower. On the usual theory we require three layers of vapour, the middle one hotter, or at any rate emitting more intense light, than the other two. There are difficulties in this theory, as we should have expected the vapour to be less dense and cooler as we recede from the photosphere. But, on the above theory, the light that appears as the lines K_2 and H_2 must be regarded as due to a high-level calcium cloud, but to have originally come from the photosphere. A spectroheliogram, therefore, taken with the second slit on the bright part of such a double reversal, would show a higher level section through the solar atmosphere than one taken with the light further away from the absorption line. Both calcium masses of these types will be, of course, self-luminous to some extent, and the bright reversal is bright and the wings of the line are dark only by contrast, but the extra brightness of the H_2 and K_2 lines is due to light originally from the photosphere. If this view of one of the possible causes of such double reversals be correct, we might expect that spectroheliograms taken with the light on the two sides of an absorption line would show different markings. Hale* has taken such photographs and has observed no notable differences, but certain small dissimilarities which may be due to various causes. It may be that it is only some of those small differences which are to be explained by dispersion effects, but in any case it seems probable that masses which we may call of the type B are not very large, and if they are not comparable with the detail observable in the spectroheliogram, then the distribution over the solar disc of light of wave-lengths corresponding to the bright parts of the reversed lines will be, as far as present instruments are concerned, practically the same. The only places where we should expect great differences to make an appearance would be near the limb of the Sun, or close to a sun-spot or vortex in the solar atmosphere. Very careful examination of photographs taken with the slit at varying distances from the H_3 K_3 lines would be necessary to detect slight differences such as might present themselves.

* *Comptes Rendus*, cxlviii. p. 1025.

If this view of the nature of some of the lines appearing in the solar spectrum be a correct one, the whole question of the motion of vapours in the line of sight is affected. In the case we have been considering, the H_2 and K_2 lines of calcium, if due to deviation of photospheric light, will not in general appear in a very definite position, and measurements of their position, or of the H_3 and K_3 lines, cannot give the velocity of the vapour in the line of sight.

The present communication is to indicate a possible cause of some of the phenomena which may operate in giving rise to some of the double reversals observed in the case of the Sun. No claim is made that all the phenomena are to be explained on these lines, but account should be taken of the facts of anomalous dispersion in looking for detailed explanations of some of the most interesting observations in solar work. We have in the disturbances that must occur in the solar atmosphere sufficient cause for the production of such systems of vapour as we have supposed. If the faculæ are regions where columns of vapour are rising into the Sun's atmosphere, these will spread out into extended clouds, and we should have layers of vapour at different levels, as we have layers of air with varying quantities of water vapour in the Earth's atmosphere. The higher-level vapours will be less dense than those of lower level in general, and this gives us all we want to explain the phenomena of the double reversal.